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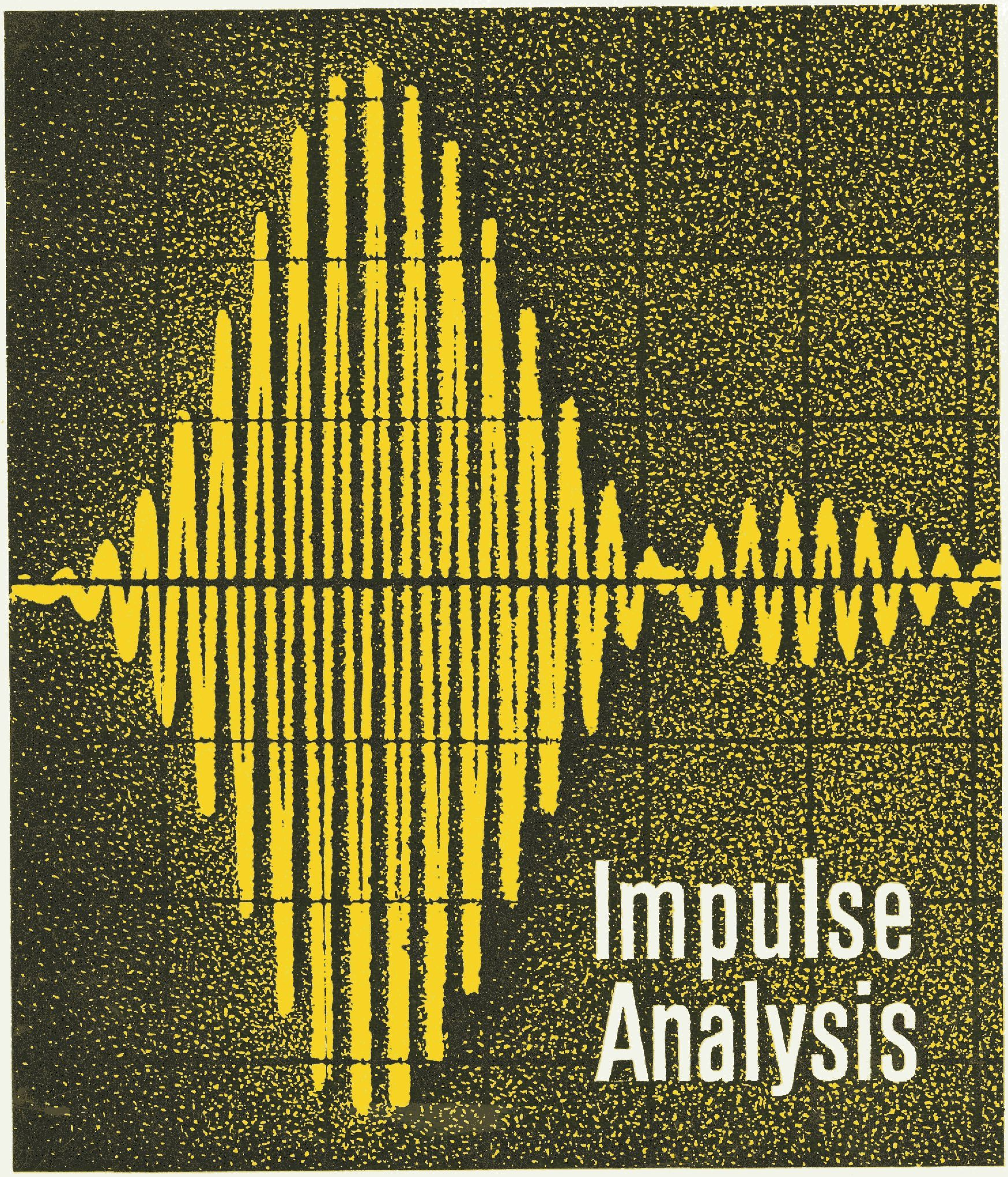

Impulse Analysis

No. 3

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TECHNICAL REVIEW

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Contents

On the Frequency Analysis of Mechanical Shocks and Single Impulses*)

by

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and

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ABSTRACT

The article points out that frequency domain descriptions (Fourier Spectra) of shocks or impulses are superior to time domain descriptions for the estimation of the responses of mechanical systems to shock loading.

The theory for filter-response to impulses is outlined and it is shown that, provided that the filter bandwidth is narrow compared to one divided by the impulse duration, the peak response of a filter to an impulse is proportional to the filter bandwidth and to the Fourier Spectrum value at the filter centre frequency. It is also shown that the squared and integrated value of the filter output is proportional to the bandwidth and to the squared Fourier Spectrum value.

The theoretical results have been verified by practical measurements.

Another method of obtaining the Fourier Spectrum, i.e. by repeating the impulse to obtain a periodic signal for line spectrum measurements, is briefly outlined and a comparison is made between the measureable values of the different methods.

SOMMAIRE

L'article souligne que l'analyse fréquentielle (Spectre de Fourier) des chocs et impulsions est mieux adaptée que l'analyse temporelle pour l'estimation de la réponse des systèmes mécaniques soumis a un choc.

La théorie de la réponse impulsionelle des filtres est esquissée, et, sous la condition que la bande passante du filtre soit faible comparée à l'inverse de la durée de l'impulsion, on montre que cette réponse est proportionelle à la bande passante et à la valeur du spectre de Fourier pour la fréquence centrale du filtre. On montre aussi que la valeur carrée et intégrée de la tension de sortie du filtre est proportionelle à la bande passante et à la valeur carrée du spectre de Fourier.

Les résultats théoriques ont été vérifiés par des mesures expérimentales.

Une autre méthode d'obtention du spectre de Fourier, répitition de l'impulsion pour obtenir un signal périodique et une mesure de spectre de raies, est brievement esquissée. Une comparaison est faite entre les valeurs mesurables pour les différentes méthodes.

ZUSAMMENFASSUNG

Im Artikel wird zum Ausdruck gebracht, daß für die Abschätzung des Verhaltens mechanischer Systeme bei Stoßbeanspruchung Beschreibungen stoßförmiger oder impulsförmiger Vorgänge in Abhängigkeit von der Frequenz (Fourierspektren) Beschreibungen in Abhängigkeit von der Zeit vorzuziehen sind.

Die Theorie der Filterantwort auf Impulse wird hervorgehoben und ferner wird gezeigt, daß unter der Voraussetzung, daß die Filterbandbreite gegenüber einer durch die Impulsdauer dividierten Bandbreite schmal ist, die Spitzenantwort eines Filters auf einen Impuls der Filterbandbreite und dem Fourierspektralwert bei der Filtermittenfrequenz proportional ist. Zusätzlich wird noch dargelegt, daß der quadrierte und integrierte Wert des Filterausganges der Bandbreite und dem quadrierten Fourierspektralwert proportional ist. Die theoretischen Ergebnisse sind durch praktische Messungen bestätigt worden.

Ein weiteres Verfahren zur Erzielung des Fourierspektrums, nämlich die Wiederholung des

*) Paper presented at the 78th Meeting of the Acoustical Society of America, San Diego, California, November 1969.

Impulses, um ein periodisches Signal für die Bestimmung des Linienspektrums zu erhalten, wird kurz behandelt, und es werden Vergleiche zwischen den nach den verschiedenen Verfahren erhältlichen Meßgrößen angestellt.

Introduction

The simplest method of describing a mechanical shock or impulse is to obtain a graphic (or photographic) record of the pulse in the *time domain,* i.e. to obtain an amplitude versus time trace of the pulse.

If the pulse is applied to a linear (mechanical) system, and the response of the system to a unit impulse (δ-impulse) is known, the response of the system to the pulse in question can be estimated by superposition:

$$X(t) = \int_{-\infty}^{t} f(\tau) \times h(t-\tau)\, d\tau$$

where $X(t)$ is the response
where $f(\tau)$ is the forcing function (impulse)
where $h(t-\tau)$ is the unit impulse response of the system
and τ is a "dummy" time variable.

As the above integral involves the convolution of two, often very complicated mathematical functions, it is readily seen that an exact solution of the integral might pose formidable difficulties.

In estimating the response of a system to impulse excitation other than the above mentioned time-domain description of the phenomena might therefore be more convenient. One such method of description is obtained by applying the *Fourier Transform* to the phenomena, which then allows for a description in the *frequency domain.*

Frequency domain description have two major advantages above time domain descriptions:

1. A frequency description of the exciting pulse shows what frequencies are important, and thus in which frequency regions dangerous resonance build-up in the response might occur.
2. Response calculations in the frequency domain normally involves multiplication only (not convolution).

The Fourier Transform is mathematically defined as

$$A(f) = \int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt$$

with the additional requirement that

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$

i.e. that $f(t)$ is finite.

When $f(t)$ is a shock or an impulse the latter requirement is automatically fulfilled.
In this paper the evaluation of the Fourier transform of shocks and impulses in practice is discussed, and some typical measuring arrangements used for the evaluation are outlined.

On the Response of "Ideal" Filters to Very Short Duration Impulses
To simplify the theoretical treatment the case of the "ideal" filter is considered in the following.

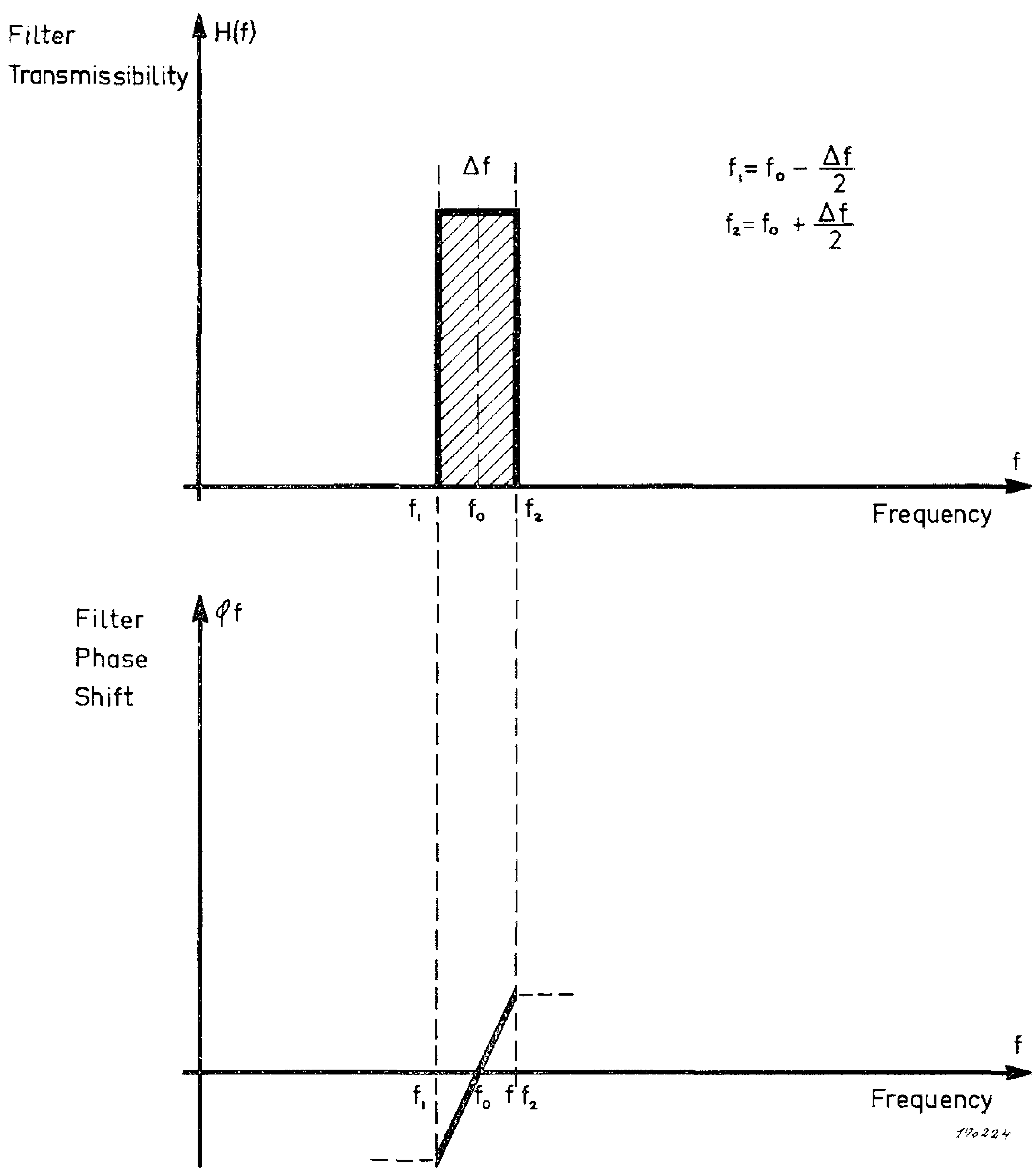

Fig. 1. Transfer function and phase shift of an "ideal" filter.

The "ideal" filter is defined as shown in Fig. 1, and is assumed to have unity gain in the pass-band, and zero elsewhere. Also, it is assumed that the phasechange within the filter bandwidth follows the relationship (Fig. 1):

$$\varphi_f = 2\pi (f - f_0)\, t_L$$

where t_L is the "transmission time" of the filter (not to be confused with the filter "ringing" time, or transient built-up). The response of such a filter to a unit impulse can be found by means of Fourier transform methods and is (see also Appendix A):

$$F(t) = 2\,\Delta f \,\frac{\sin[\pi\,\Delta f\,(t - t_L)]}{\pi\,\Delta f\,(t - t_L)}\,\cos(2\,\pi\,f_o\,t)$$

where Δf is the filter bandwidth and f_o is its center frequency. The function, $F(t)$, is plotted in Fig. 2, and the meaning of the "transmission time" is clearly noticed from the figure. Also, when $f_o \gg \Delta f$ it can be seen from the figure that the maximum response (peak response) of the filter occurs when $t = t_L$ and is thus simply

$$F_{max}(t) = 2\,\Delta f$$

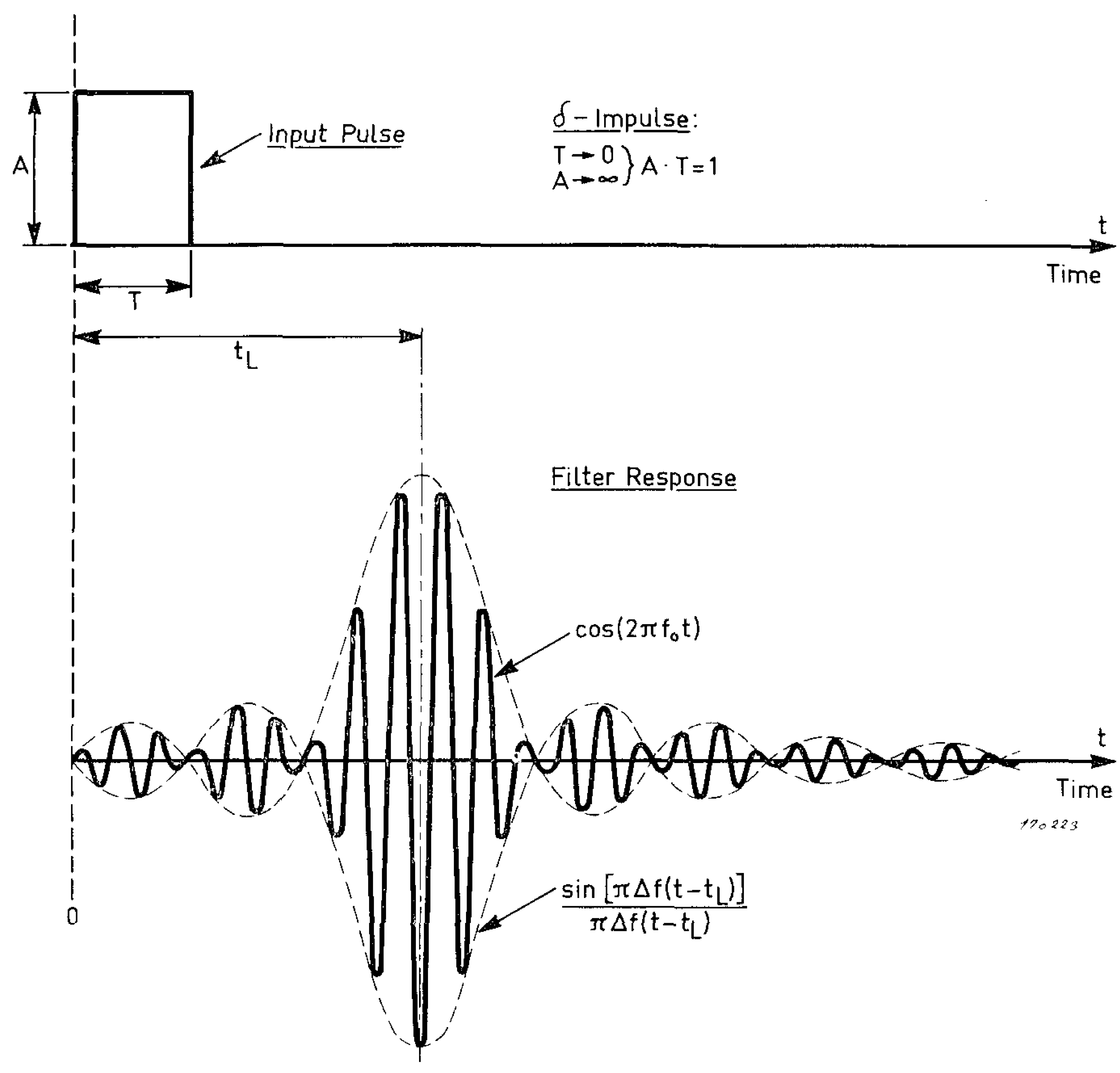

Fig. 2. Response of the "ideal" filter to a unit-impulse.

Thus, the *peak response of the filter is proportional to the filter bandwidth.* The energy, or rather the squared and integrated, response of the filter can be found by evaluating the integral

$$E = \int_{-\infty}^{\infty} F^2(t)\,dt = 4\,\Delta f^2 \int_{-\infty}^{\infty} \left[\frac{\sin[\pi\,\Delta f\,(t - t_L)]}{\pi\,\Delta f\,(t - t_L)}\right]^2 \cos^2(2\,\pi\,f_o\,t)\,dt$$

As

$$\cos^2 (2\,\pi\, f_o\, t) = \frac{1}{2} [\cos (4\,\pi\, f_o\, t) + 1]$$

then
$$E = 2\,\Delta f^2 \int_{-\infty}^{\infty} \left[\frac{\sin [\pi\, \Delta f\, (t - t_L)]}{\pi\, \Delta f\, (t - t_L)} \right]^2 \cos (4\,\pi\, f_o\, t)\, dt$$

$$+ 2\,\Delta f^2 \int_{-\infty}^{\infty} \left[\frac{\sin [\pi\, \Delta f\, (t - t_L)]}{\pi\, \Delta f\, (t - t_L)} \right]^2 dt$$

One way of obtaining an estimate of these integrals is to set $t_L = 0$, integrate the expression from 0 to ∞ and multiply the results by two, see also Fig. 2.

From H. B. Dwight's "Tables of Integrals and Other Mathematical Data" it is found that integrals of the type:

$$\int_0^{\infty} \frac{\sin^2 (ax) \cos (mx)}{x^2}\, dx$$

are equal to zero when $\frac{m}{2} \geqq a \geqq 0$. The first of the two integrals above satisfy these conditions so that:

$$E = 2 \times 2\,\Delta f^2 \int_0^{\infty} \left(\frac{\sin (\pi\, \Delta f\, t)}{\pi\, \Delta f\, t} \right)^2 dt$$

$$= \frac{4}{\pi^2} \int_0^{\infty} \left(\frac{\sin (\pi\, \Delta f\, t)}{t} \right)^2 dt$$

This integral is of the type

$$\int_0^{\infty} \frac{\sin^2 (mx)}{x^2}\, dx = | m | \times \frac{\pi}{2}$$

Thus

$$E = \frac{4}{\pi^2}\, \pi\, \Delta f \times \frac{\pi}{2} = 2\,\Delta f$$

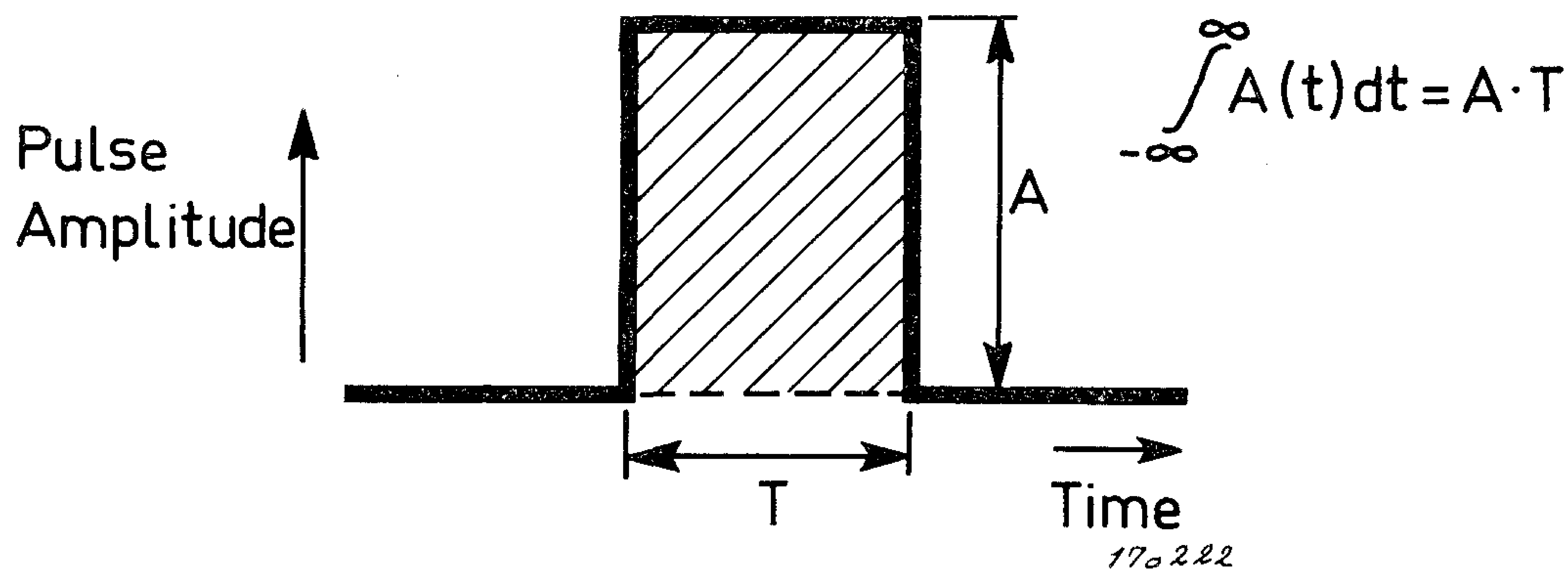

Fig. 3. Illustration of characteristic quantities describing a rectangular pulse.

If the impulse applied to the filter is no longer a unit impulse, but still *very* short compared to 1/Filter Bandwidth (i.e. $T \ll \frac{1}{\Delta f}$) then

$$F_{max}(t) = 2 A T \Delta f \times F_o(f)$$

$$E = \int_{-\infty}^{\infty} F^2(t)\, dt = 2 A^2 T^2 \Delta f \times F_o^2(f)$$

where AT is the amplitude-time integral of the pulse (see also Fig. 3 and Appendix A), and $F_o(f)$ is a frequency weighting function to be explained below.

The above results for *very short impulses* show a very interesting *fact, namely that both the peak amplitude and the squared and integrated response of the filter have the same relationship to filter bandwidth.* Thus if the pulse spectrum is measured with a set of constant percentage bandwidth filters the measured squared and integrated value as well as the peak amplitude increases linearily with frequency.

Another interesting fact, which is further discussed below, is that as long as the requirement ($T \ll \frac{1}{\Delta f}$) is fulfilled, and the pulse spectrum is measured by means of *constant bandwidth filters the measured result bears a simple direct relationship to the Fourier spectrum of the pulse.* If T becomes of the order of $1/\Delta f$, or larger, this relationship is upset.

Discussion of Pulse Measurements in the Frequency Domain

It was stated in the introduction that frequency domain descriptions of single pulses can be obtained mathematically by means of the Fourier transform. Examples of such frequency domain descriptions (Fourier spectra) of pulses

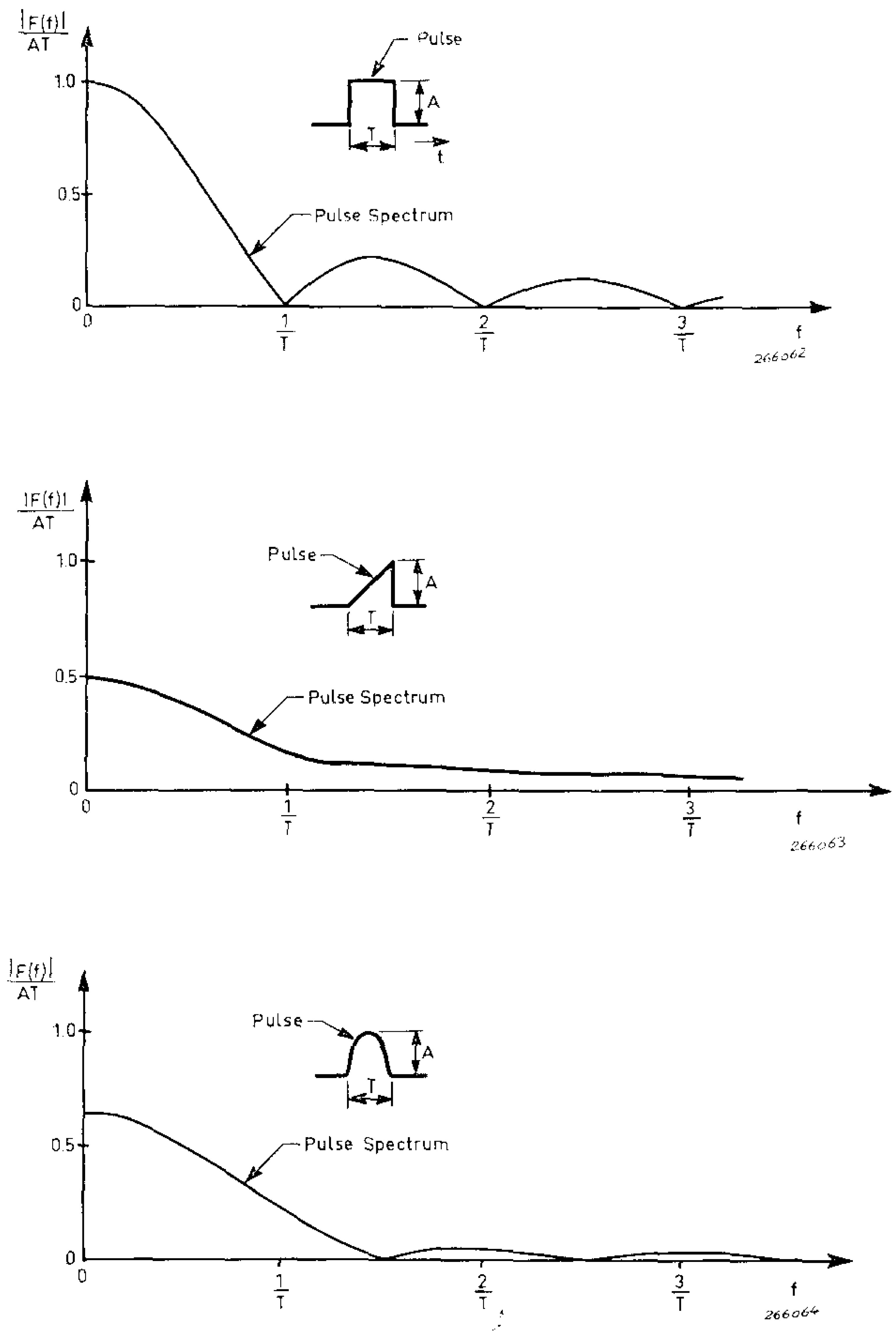

Fig. 4. Fourier spectra for various pulses.

with different shapes are shown in Fig. 4. Although the spectrum of the rectangular pulse is used in the following to demonstrate the effect of filtering, the principles discussed are applicable to any pulse spectrum.

If the pulse is applied to an ideal filter with the bandwidth, Δf, only frequencies which are inside Δf will be transmitted through the filter. The effect of filtering is illustrated in Fig. 5 for the case when $\Delta f \ll \frac{1}{T}$ (which is the same condition as mentioned above, $T \ll \frac{1}{\Delta f}$). It is clear that what is measured at the output of the filter must be directly related to that part of the pulse spectrum which is inside Δf, i.e. it must be a measure of the pulse Fourier spectrum at (and around) the frequency f_o.

Now the Fourier spectrum of the rectangular pulse is given mathematically by the expression:

$$A(f) = A\,T \frac{\sin(\pi f T)}{\pi f T}$$

By multiplying this expression by the frequency response function of the (ideal) filter and taking the inverse Fourier transform (see Appendix A) of the result the time function of the filter output can be obtained:

$$F(t) = 2 \int_0^\infty A\,T \frac{\sin(\pi f T)}{\pi f T} \times \cos\left[2\pi f (t - t_L) + 2\pi f_o t_L\right] df$$

$$= 2 \int_{f_o - \frac{\Delta f}{2}}^{f_o + \frac{\Delta f}{2}} A\,T \frac{\sin(\pi f T)}{\pi f T} \times \cos\left[2\pi f (t - t_L) + 2\pi f_o t_L\right] df$$

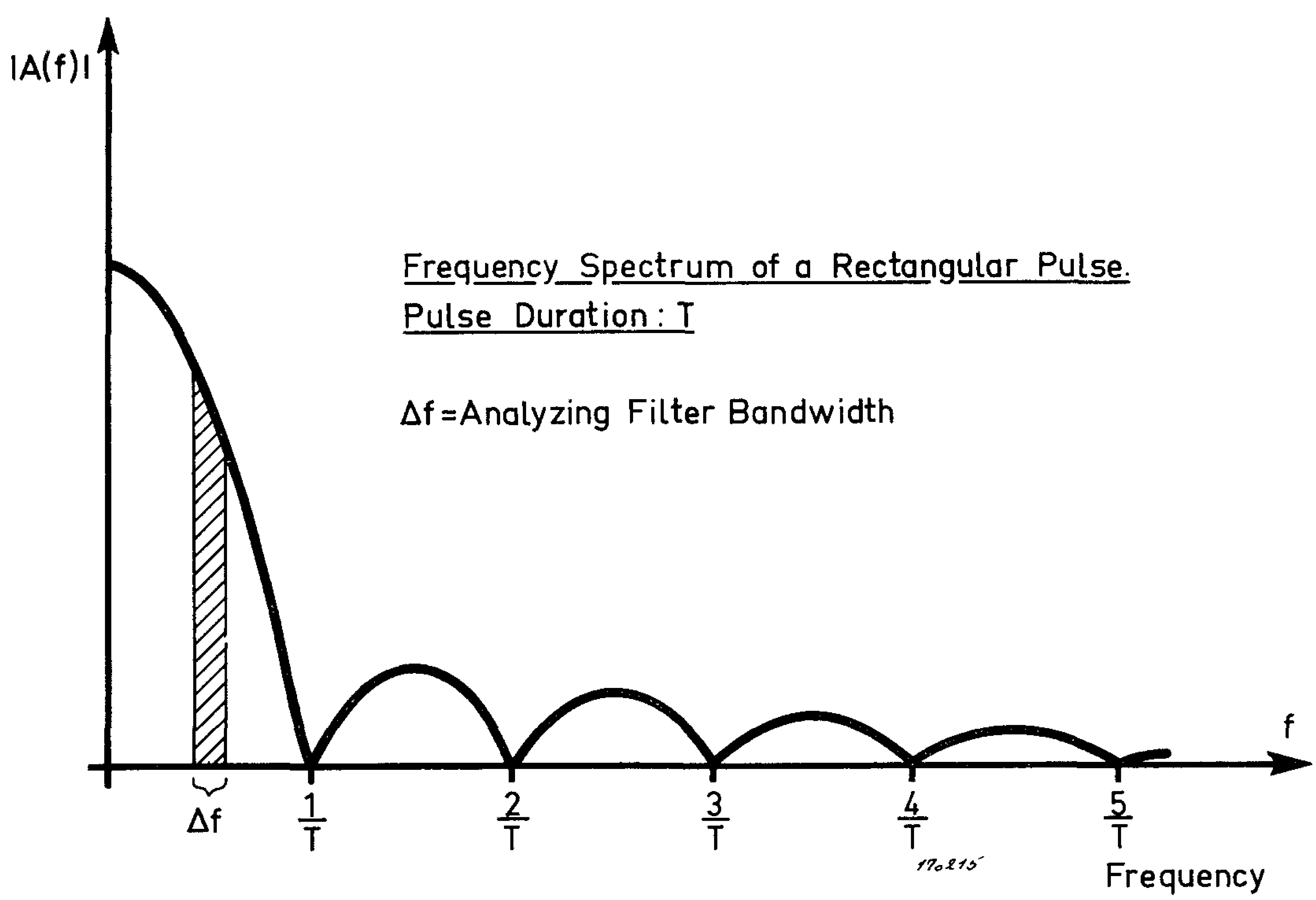

Fig. 5. Effect of narrow band filtering of a pulse in the frequency domain

Assuming that the value of $\frac{\sin(\pi f T)}{\pi f T}$ changes so little in the interval $f_o - \frac{\Delta f}{2}$ to $f_o + \frac{\Delta f}{2}$ that it may be considered "constant" within this range no large error is made when this factor is placed outside the integral, thus

$$F(t) \approx 2\,A\,T\,\frac{\sin(\pi f_o T)}{\pi f_o T}\int_{f_o-\frac{\Delta f}{2}}^{f_o+\frac{\Delta f}{2}} \cos\left[2\pi f(t-t_L)+2\pi f_o t_L\right] df$$

Actually, this assumption corresponds to the requirement for the use of very narrow band filters, and is graphically illustrated in Fig. 6.

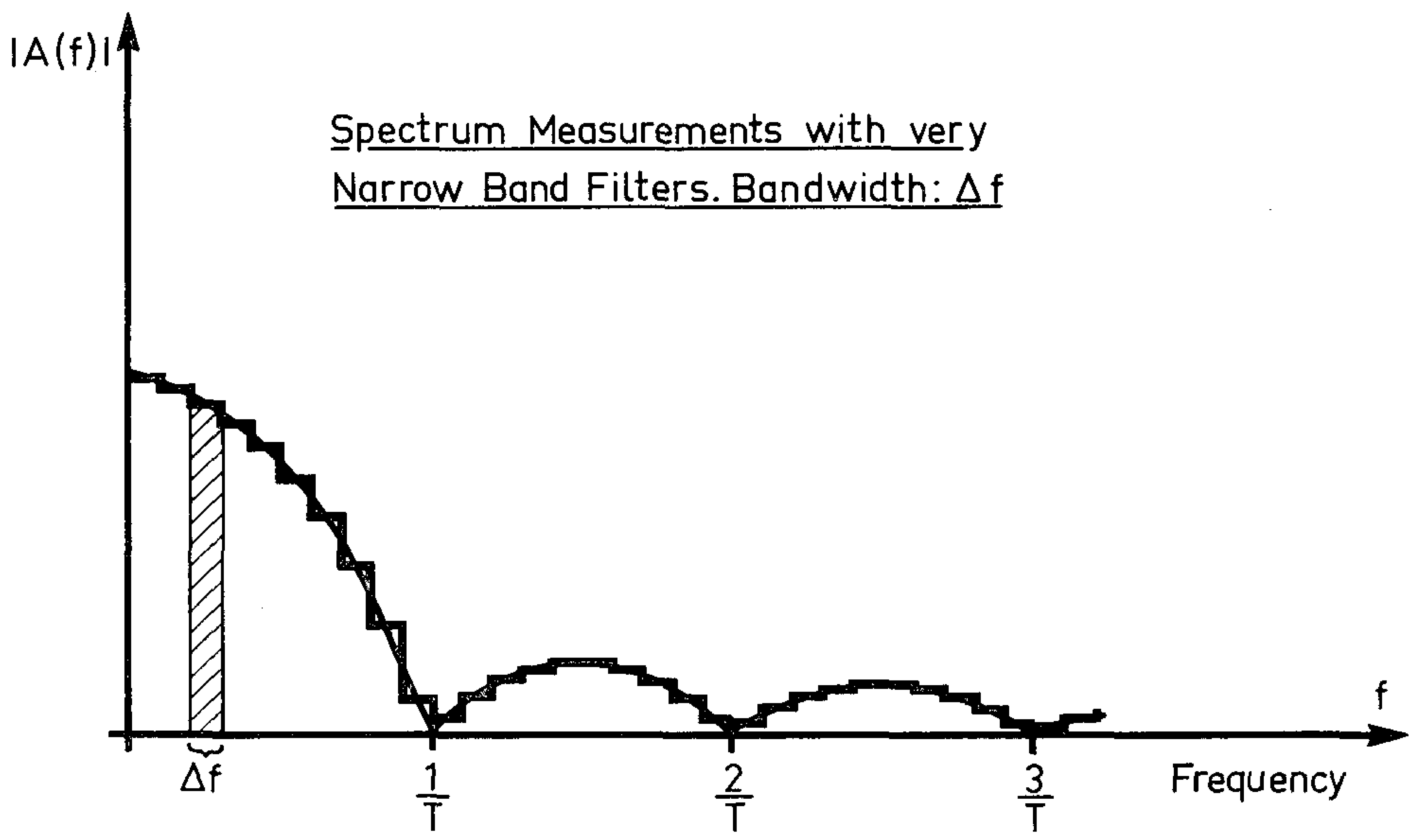

Fig. 6. Spectrum measurements. Note: At minimas ($f = n/T$) the filter responses are not directly corresponding to the Fourier spectrum-curve.

Solving the above integral gives:

$$F(t) \approx 2\,A\,T\,\frac{\sin(\pi f_o T)}{\pi f_o T}\times \Delta f \times \frac{\sin[\pi\,\Delta f\,(t-t_L)]}{\pi\,\Delta f\,(t-t_L)}\cos(2\pi f_o t)$$

Calculating now the peak response as well as the squared and integrated response of the filter in exactly the same way as was done previously for the unit impulse one obtains:

$$\underline{F_{max}(t) = 2\,A\,T\,\frac{\sin(\pi f_o T)}{\pi f_o T}\times \Delta f}$$

and

$$\underline{E = 2\,A^2\,T^2\left(\frac{\sin(\pi f_o T)}{\pi f_o T}\right)^2\times \Delta f}$$

Thus a good *approximation* of the Fourier spectrum value at f_0 can be obtained either by measuring the peak response of the filter to the pulse and dividing it by two times the filter bandwidth ($2\,\Delta f$), or by measuring the squared and integrated output from the filter, dividing it by $2\,\Delta f$ and extracting the squareroot of the result.

Considering the above it is also readily seen why *measured results obtained with filters the bandwidth of which are of order of, or larger than 1/T (T = pulse duration), cannot be used to determine the Fourier spectrum of the pulse.* Graphically two such cases are exemplified in Fig. 7. To further illustrate what is actually measured in the two cases shown in Fig. 7 it might

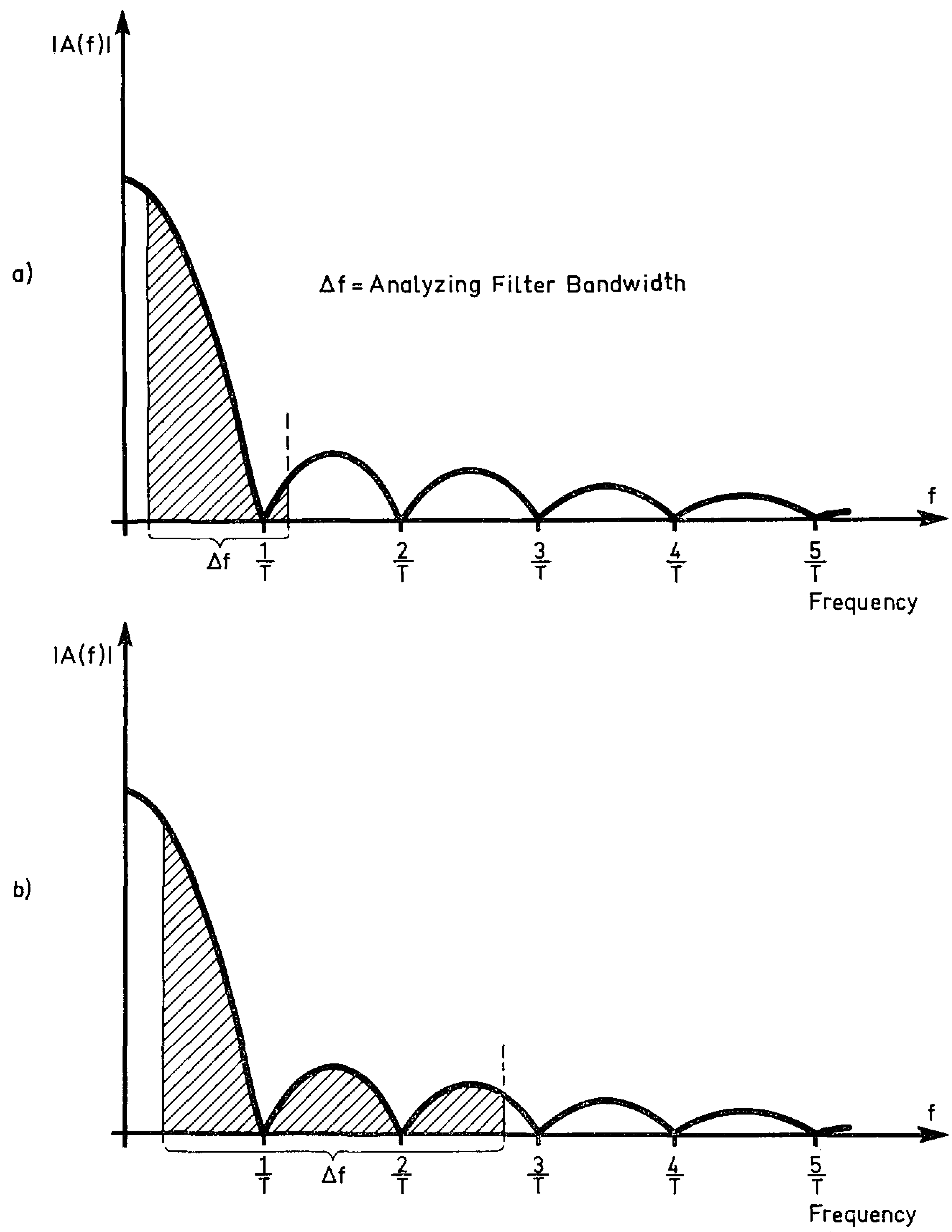

Fig. 7. Effect of broad-band filtering.

be of interest to see what the corresponding time function outputs from the filter would look like. This is sketched in Fig. 8, and it is seen that instead of *one* (oscillating) "pulse" (cfr. Fig. 2) the filter output looks like *two* (oscillating) "pulses", one which is caused by the response of the filter to the "start" of the original pulse and one which is caused by its sudden ceasation. In other words: The filter responds, no longer to a pulse, but more or less to two *step* functions, the distance between the two steps being equal to the original pulse duration.

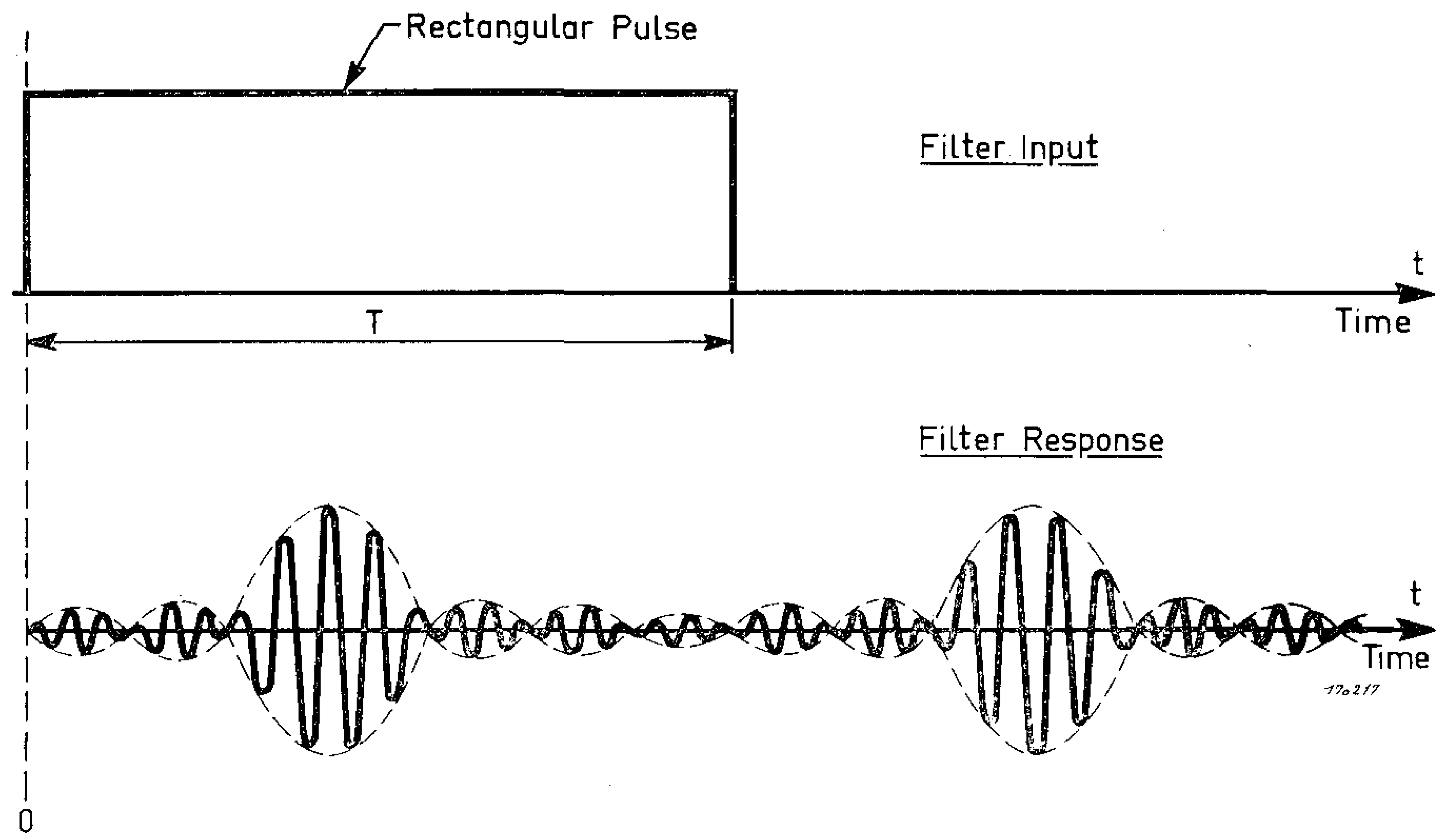

Fig. 8. Broad-band filter-response to a pulse.

Practical Frequency Analysis of Single Impulses

Fig. 9a shows the result of a practical measurement obtained by filtering, squaring and integrating, while the theoretical Fourier spectrum for the same pulse is shown in Fig. 9b). The pulse duration was here 60 milliseconds and the bandwidth of the analyzing filter was 3,16 Hz. The practical measuring arrangement used to obtain the result Fig. 9a) is sketched in Fig. 10. It consists of an FM magnetic Tape Recorder, an electronic gating circuit, a Heterodyne Slave Filter with associated Amplifier and Tuning Oscillator, a Multiplier (Squarer) with integrating circuitry and a Level Recorder. The Tape Recorder is supplied with an endless loop on which the pulse to be analyzed is recorded, and the electronic gating circuitry ensures that the output signal from the Tape Recorder is zero except for a certain interval around the pulse, Fig. 11. If the gating circuit was not used possible extraneous noise signals would decrease the measuring accuracy. Actually, the gating circuit is also used to reset the integrator to zero after the pulse has been measured, resulting in a recording of the type shown in Fig. 9a).

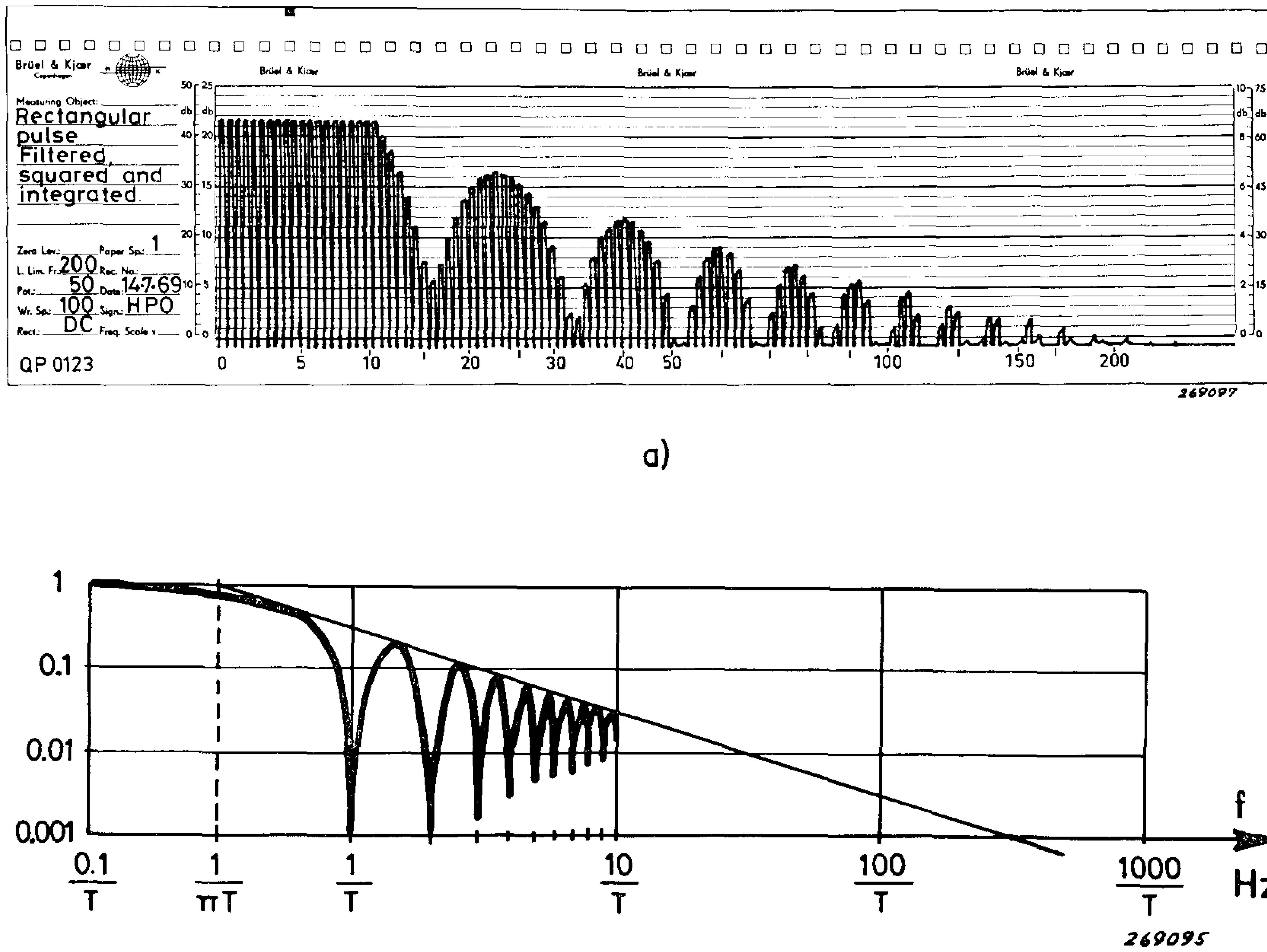

Fig. 9.

a) A practical recording of the squared and integrated filter output when a rectangular pulse is applied once to the filter input for each filter centre frequency.

b) The theoretical Fourier spectrum for a rectangular pulse.

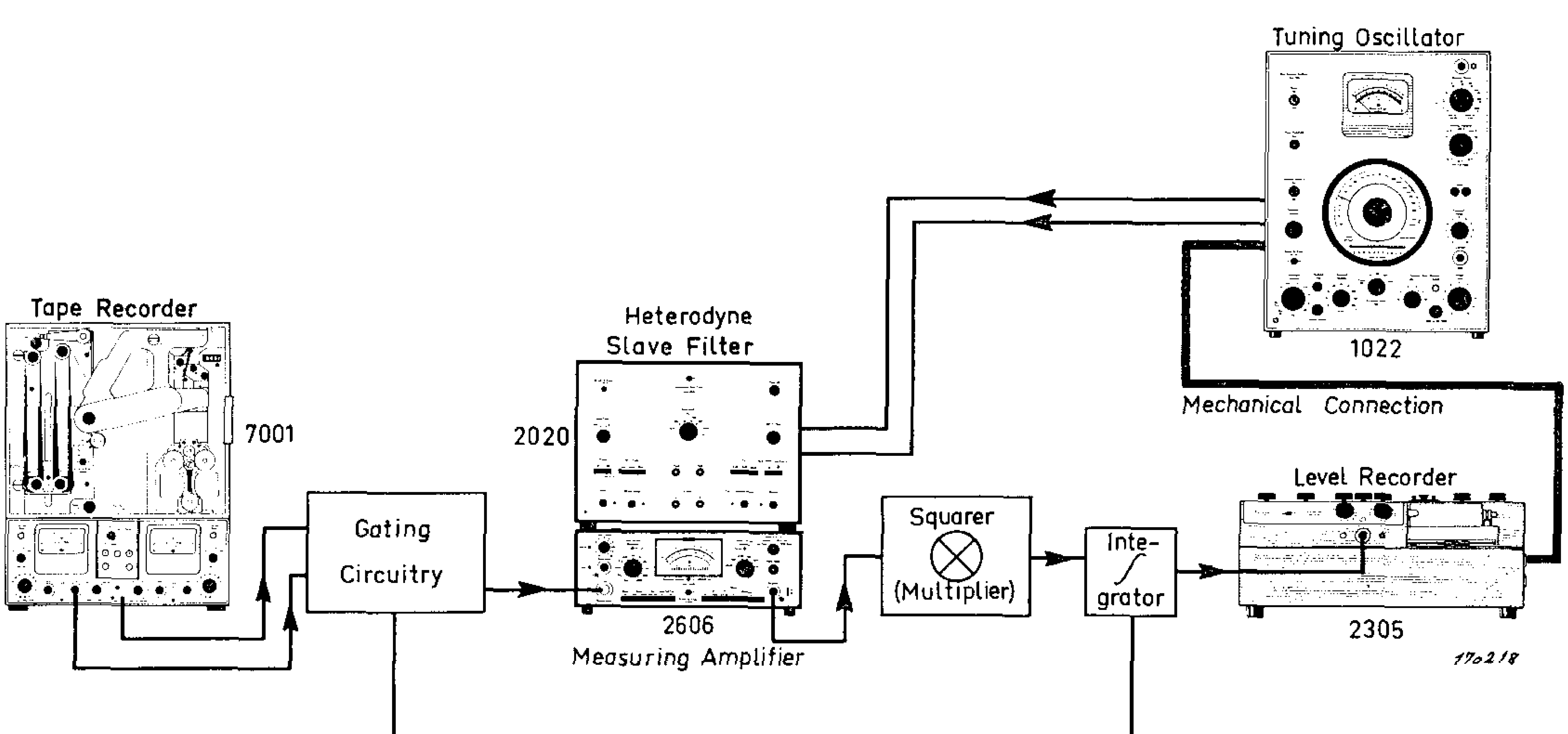

Fig. 10. The measuring arrangement used to obtain the recording shown in Fig. 9a.

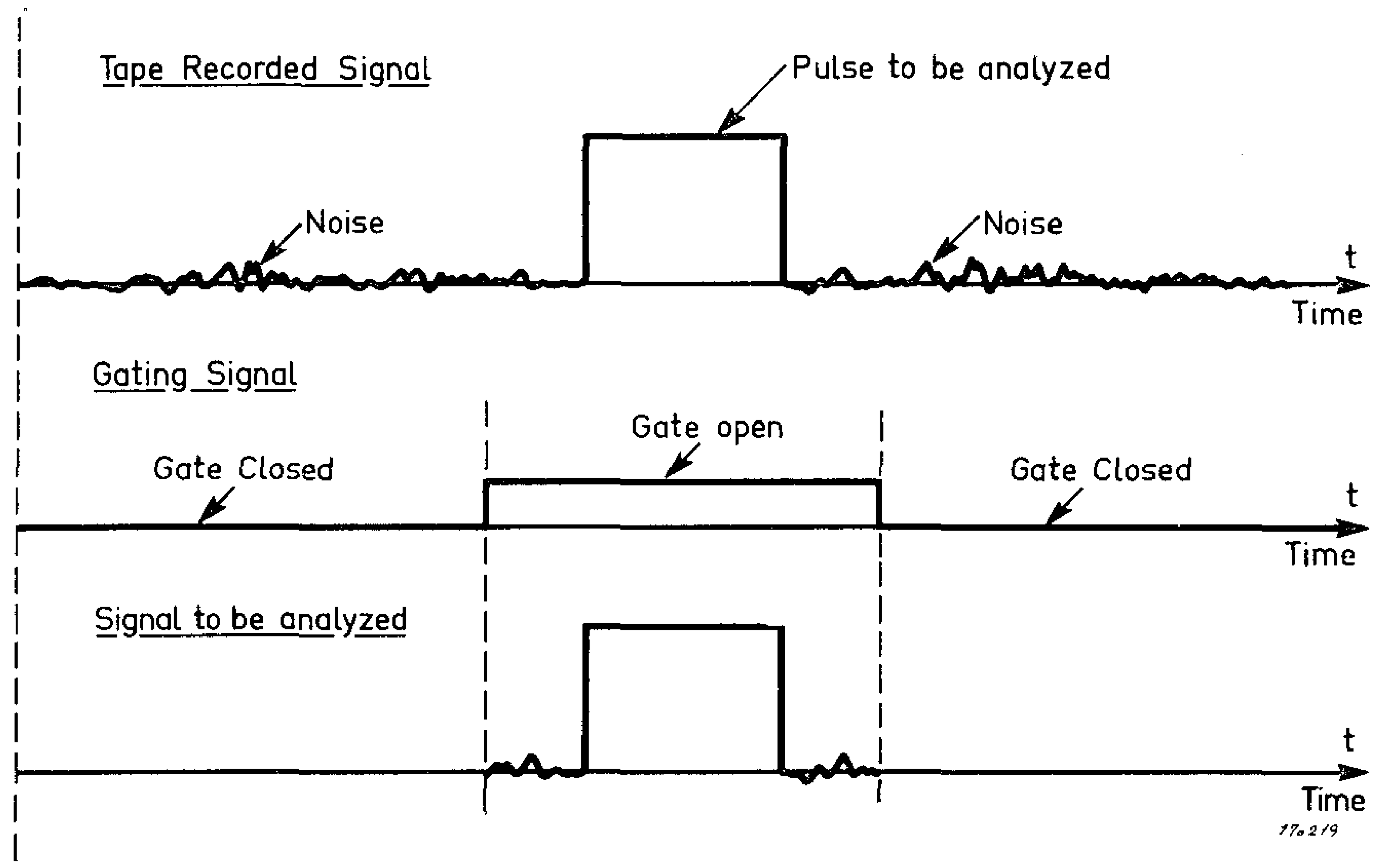

Fig. 11. Illustration of the effect of electronic gating.

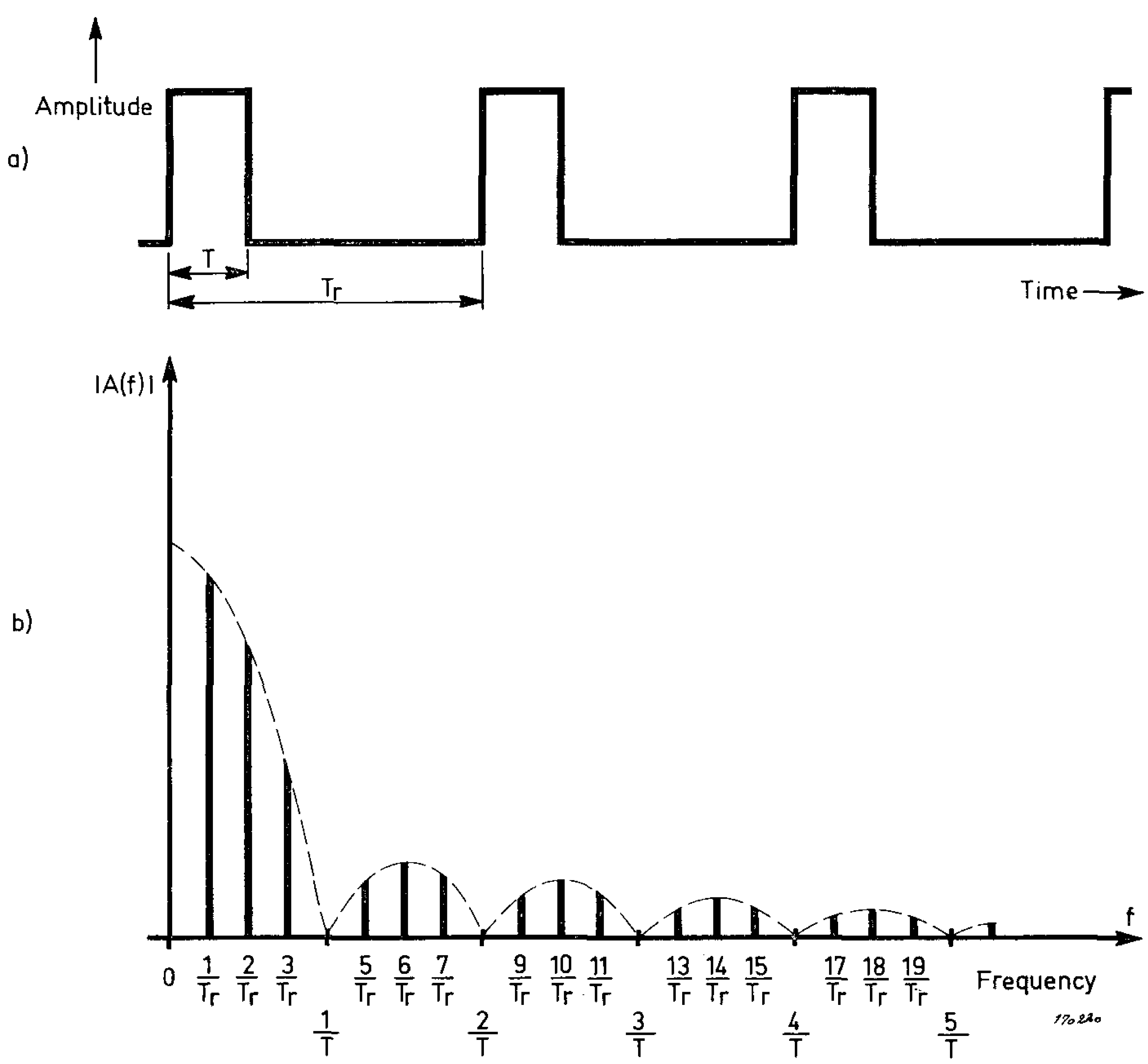

Fig. 12. A pulse train and its Fourier line-spectrum.

Allthough the above described measurement method may be the most straightforward method to obtain an estimate of the Fourier spectrum of a pulse, other methods do exist. One such method, commonly used in practice, consists in transforming the pulse into a continuous pulse train, i.e. repeating the pulse periodically, and analyzing the pulse train in the manner normally used for the frequency analysis of periodic signals. In this way a so-called line-spectrum is obtained which consists of discrete frequency components. The amplitude of each component is a measure of the Fourier spectrum at that particular frequency, as shown in the following:
If the period of repetition of the pulse is T_r, (Fig. 12a), then, from the theory of Fourier series, it is known that each component in the series is harmonically related to the fundamental frequency $1/T_r$ as shown in Fig. 12b). Fig. 12b) shows the Fourier spectrum of a series of rectangular pulses with a pulse duration T. It is seen that when $T_r \gg T$ a great number of discrete frequency components exists, and in the limit when $T_r \rightarrow \infty$ the Fourier spectrum of the single pulse may be obtained. That this is so can also be readily shown mathematically, and the derivation is found in any text-book on pulse analysis.

However, as the magnitude of the various harmonics is normally found by means of a time-averaging process (RMS) it can also be seen from Fig. 12a) that when T_r becomes much larger than T the magnitude of the various frequency components becomes smaller (actually when $T_r \rightarrow \infty$ the magnitude of the discrete components tends towards zero). The Fourier spectrum of the single pulse must therefore be defined in terms of *spectral density* rather than in terms of magnitude where *spectral density is defined as a magnitude measure per unit frequency.*
From a practical measurement point of view it is very important to choose the ratio T_r/T correctly when the method of periodic repetition of the pulse, and time averaging, is used.
Considering again the Fourier spectrum of a (periodically repeated) rectangular pulse it is seen that the zeros in the theoretical pulse spectrum occur with frequency intervals of $1/T_r$. To be able to obtain more than one spectral line between successive minima the ratio T_r/T must therefore be larger than two. Experiments have shown that some 5 lines between minima seem to give a sufficiently good resolution of the spectrum. On the other hand, the larger ratio T_r/T becomes, the smaller becomes the available dynamic range for the analysis. A too large ratio T_r/T must also be avoided due to crest-factor limitations in the measuring and analyzing equipment. As a practical compromise a T_r/T-ratio between 3 and 5 is recommended.
Fig. 13 shows a practical measurement arrangement used to analyze a pulse which is being repeated periodically. Again the pulse was recorded on an FM magnetic tape recorder and the tape, containing the pulse, made into an endless loop. The tape loop must, however, in this case be made very small to achieve a sufficiently high pulse repetition frequency.

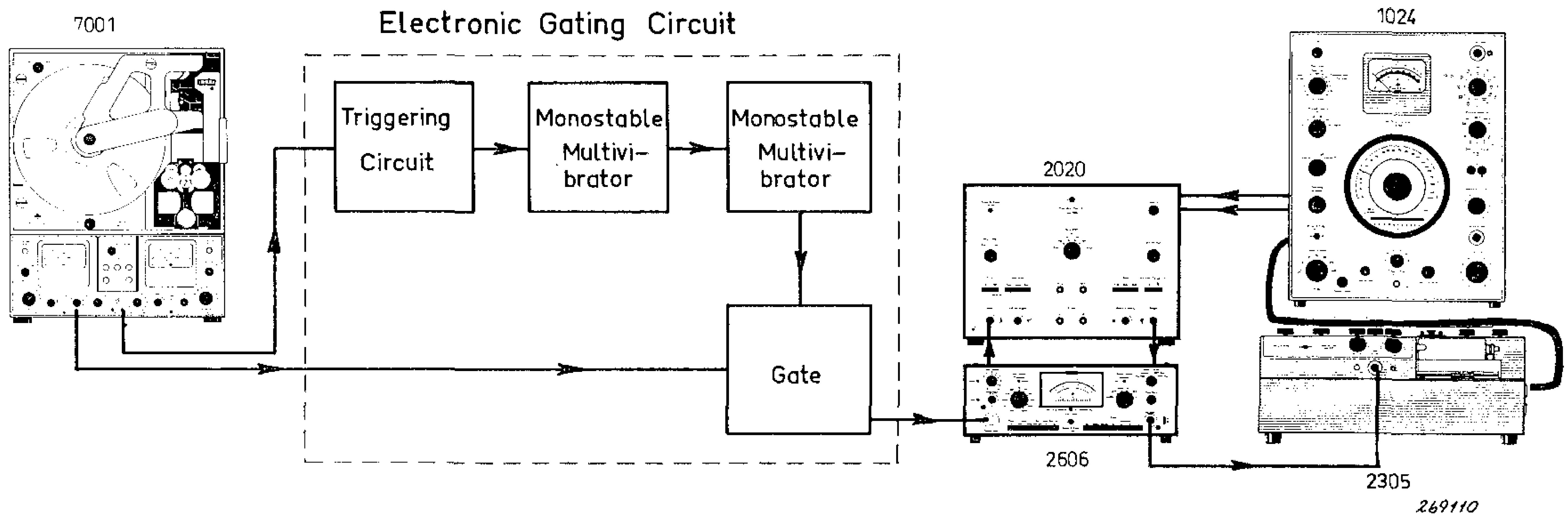

Fig. 13. Measuring arrangement used to analyse a periodically repeated pulse.

In Fig. 14 the result of an actual frequency analysis of a periodically repeated rectangular pulse is shown. The bandwidth of the analyzing filter was 3.16 Hz, and the repetition frequency of the pulse was 4.2 Hz. To obtain a pulse repetition frequency of 4.2 Hz use was made of a Tape Recorder (B & K Type 7001) running at a speed of 60" per second and supplied with a special loop adaptor which allowed the accommodation of a tape loop with a total length of 362 mm. It can be seen from the figure (Fig. 14) that with an analyzer bandwidth of 3.16 Hz the pulse repetition frequency of 4.2 Hz was just large enough for the individual lines in the spectrum to be resolved. It was, on the other hand, the highest repetition frequency which could be obtained on the Tape Recorder by relatively simple means.

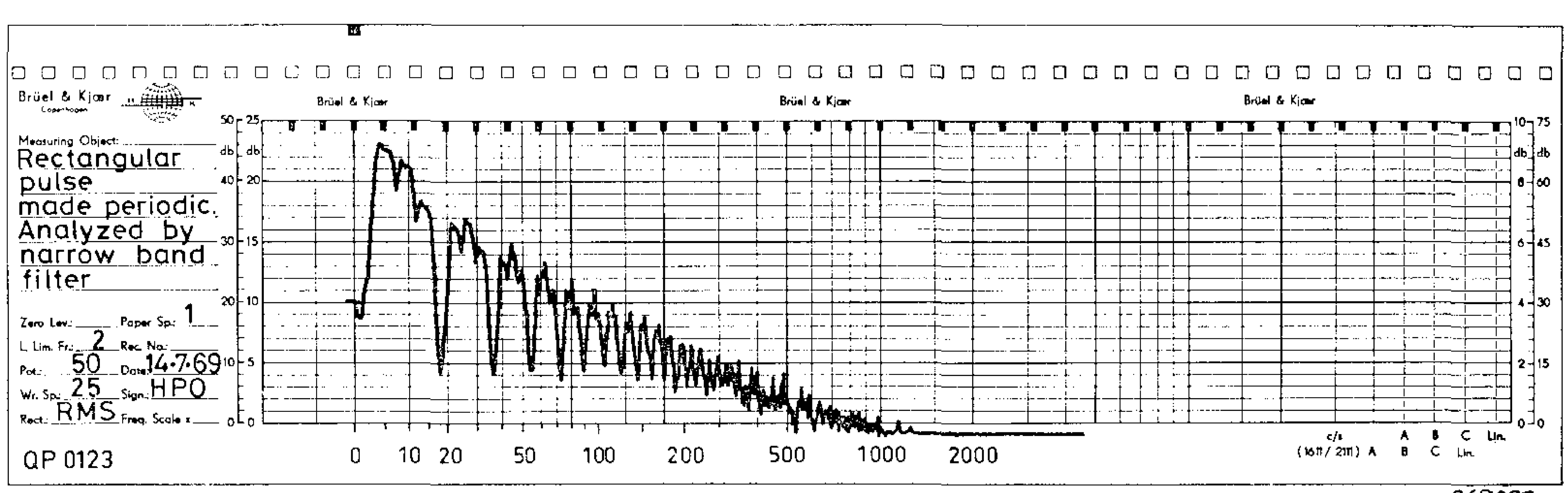

Fig. 14. Recording of the line-spectrum of a periodically repeated pulse as obtained by the measuring arrangement shown in Fig. 13.

One problem which remains when a recording of the type shown in Fig. 14 has been obtained is to calibrate the Y-axis of the recording in terms of the theoretical Fourier spectrum values. As the values actually recorded are RMS-values of the "harmonics" of the periodically repeated pulse, and the pulse Fourier spectrum is given in terms of spectral densities, then how are these values related to each other?

To solve this problem consider Fig. 15. In Fig. 15a) a repetitive signal is shown, and from the theory of Fourier series, it is known that the coefficients in the series can be found from integrals of the type:

$$C_n = \frac{2}{T_r} \int_{-\frac{T_r}{2}}^{\frac{T_r}{2}} f(t) \times e^{-j2\pi n f_0 t}\, dt$$

where f_o is the fundamental frequency and $n \times f_o$, are harmonics of this frequency. As the signal shown in Fig. 15a) only contribute to the integral during the period of time from $-\frac{T}{2}$ to $\frac{T}{2}$ the above equation can also be written:

$$C_n = \frac{2}{T_r} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \times e^{-j2\pi n f_0 t}\, dt$$

Now, for a particular frequency $f = n \times f_o$ the Fourier transform of a single impulse of duration T is given by the integral (Fig. 15b):

$$F\ f) = \int_{-\infty}^{\infty} f(t) \times e^{-j2\pi f t}\, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \times e^{-j2\pi f t}\, dt$$

By comparing the two expressions it is readily seen that at frequencies where $f = n \times f_o$ then

$$C_n = \frac{2}{T_r} \times F(n \times f_o) = \frac{2}{T_r} \times F(f)$$

As C_n represents the peak value of a sinusoidal signal, and the values measured by means of the arrangement shown in Fig. 13 are RMS values, the relationship between the theoretical $F(f)$-value and the measured RMS-value, C_{RMS}, must be:

$$F(f) = \frac{T_r}{\sqrt{2}} C_{RMS} \qquad f = n \times f_o$$

The Value of the theoretical Fourier spectrum at the frequency $f = n \times f_o$ can therefore be found simply by *multiplying the measured RMS-value with the period of repetition (in seconds) and dividing the result by squareroot two.*

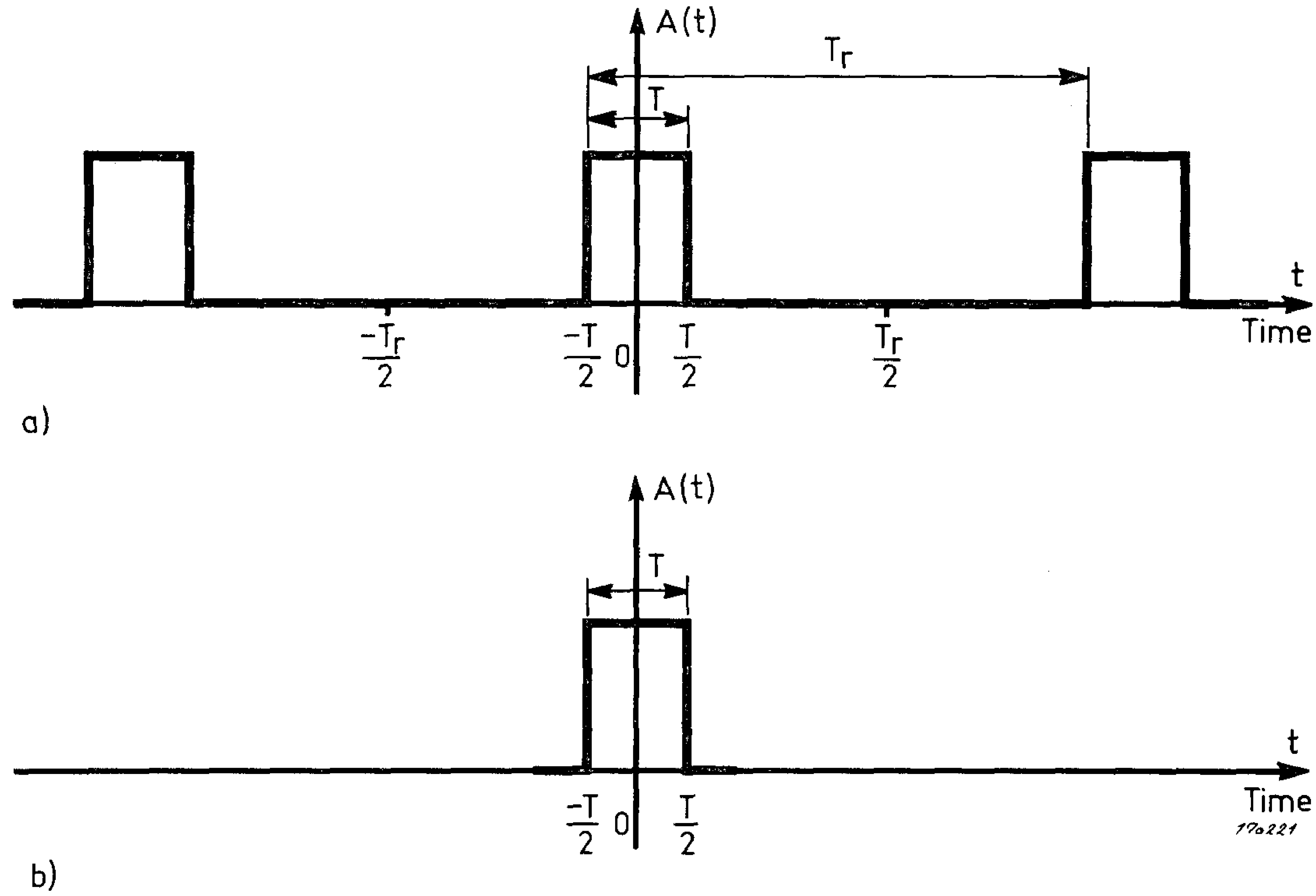

Fig. 15. A pulse train and the corresponding single pulse.

Conclusion

In the preceeding text various aspects of the practical frequency analysis of mechanical shocks and single impulses have been discussed. Allthough much of the theoretical material may be found in textbooks on Fourier transform methods it has been attempted here to give a "unified" picture of the major problems facing the engineer when he is trying to utilize the theory in practical measurements.

Appendix A

Response of an "Ideal" Filter to a Unit Impulse

To find the response of an "ideal" filter (Fig. 1 of the Text) to a unit impulse the response is first considered in the frequency domain. Use is then made of the inverse Fourier transform resulting in the desired time domain description.

A unit impulse is defined by the integral:

$$\lim_{\varepsilon \to 0} \int_{-\varepsilon}^{+\varepsilon} \delta(t)\, dt = 1$$

and is represented in the time domain by an infinitely high and infinitely narrow impulse centered around $t = 0$.

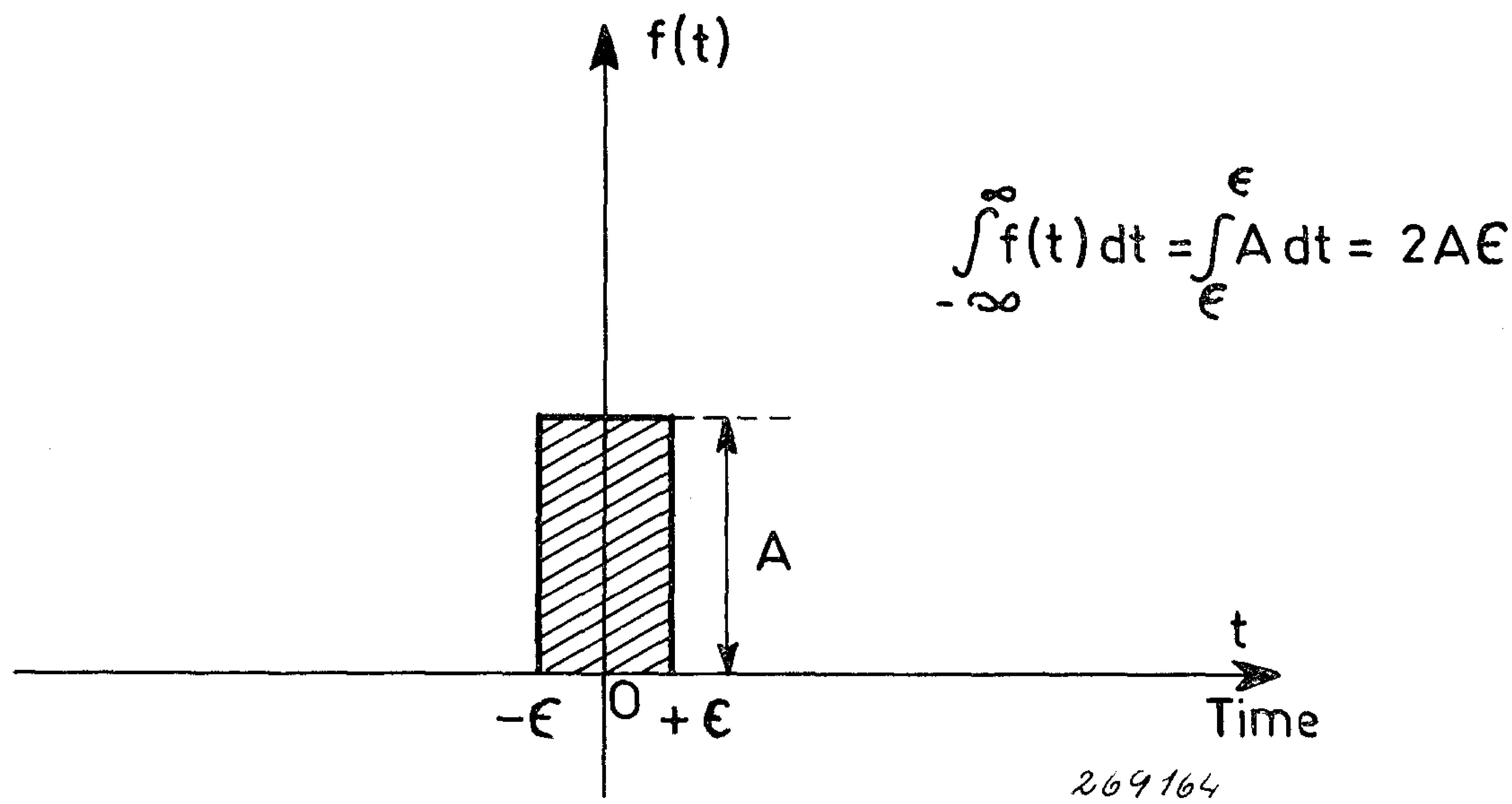

Fig. A.1. A short duration rectangular pulse.

Starting with a finite impulse, Fig. A.1, with the height A and the width 2ε it is seen that

$$\int_{-\varepsilon}^{\varepsilon} f(t)\, dt = 2A\varepsilon = \text{Time-integral (area) of the pulse}$$

where $f(t) = A$ from $-\varepsilon$ to ε and zero elsewhere. If $f(t)$ should represent a

δ-impulse then

$$\lim_{\varepsilon \to o} \int_{-\varepsilon}^{\varepsilon} f(t)\, dt = \lim_{\substack{\varepsilon \to o \\ A \to \infty}} (2 A \varepsilon) = 1$$

Taking the Fourier transform of the impulse shown in Fig. A.1 one obtains the frequency spectrum of the pulse:

$$A(f) = \int_{-\infty}^{\infty} f(t) \times e^{-j2\pi ft}\, dt = 2 A \int_{0}^{\varepsilon} \cos(2 \pi f t)\, dt$$

$$= 2 A \frac{\sin(2 \pi f \varepsilon)}{2 \pi f} = 2 A \varepsilon \frac{\sin(2 \pi f \varepsilon)}{2 \pi f \varepsilon}$$

To obtain the frequen cy spectrum of the unit impulse the above described limiting process should be applied:

$$A(f)_\delta = \lim_{\substack{\varepsilon \to o \\ A \to \infty}} \left(2 A \varepsilon \frac{\sin(2 \pi f \varepsilon)}{2 \pi f \varepsilon} \right) = 1$$

Thus the frequency spectrum of the unit impulse is constant, independent of frequency, and equal to unity.
Now, the frequency (or rather the transfer characteristic) of the "ideal" filter is unity from $f_o - \frac{\Delta f}{2}$ to $f_o + \frac{\Delta f}{2}$ and zero elsewhere. Also the phase-shift in the filter is $\varphi_f = 2 \pi (f - f_o)\, t_L$ (see text). Mathematically formulated therefore:

$$H(f) = 1 \times e^{-j\varphi} = 1 \times e^{-j2\pi(f-f_0)t} \qquad f_o - \frac{\Delta f}{2} < f < f_o + \frac{\Delta f}{2}$$

where $H(f)$ is the complex frequency response of the filter.

The frequency spectrum at the output of the filter $G(f)$ is obtained by multiplying $A(f)$ by $H(f)$:

$$G(f) = A(f) \times H(f)$$

and finally the time domain description of the filter can be found by applying the inverse Fourier transform to $G(f)$:

$$F(t) = \int_{-\infty}^{\infty} G(f) \times e^{j2\pi ft}\, df = \int_{-\infty}^{\infty} A(f) \times H(f) \times e^{j2\pi ft}\, df$$

$$= 2 \int_{0}^{\infty} Re\,[A(f) \times H(f) \times e^{j2\pi ft}]\, df$$

$$= 2 \int_{f_o - \frac{\Delta f}{2}}^{f_o + \frac{\Delta f}{2}} Re\,[1 \times 1 \times e^{-j2\pi(f-f_0)t} \times e^{j2\pi f t}]\, df$$

$$= 2 \int_{f_o - \frac{\Delta f}{2}}^{f_o + \frac{\Delta f}{2}} \cos\,[2\,\pi\,f\,(t - t_L) + 2\,\pi\,f_o\,t_L]\, df$$

Performing the integration one obtains:

$$F\,(t) = 2 \left[\frac{\sin\,[2\,\pi \left(f_o + \frac{\Delta f}{2} \right) (t - t_L) + 2\,\pi\,f_o\,t_L]}{2\,\pi\,(t - t_L)} - \frac{\sin\,[2\,\pi \left(f_o - \frac{\Delta f}{2} \right) (t - t_L) + 2\,\pi\,f_o\,t_L]}{2\,\pi\,(t - t_L)} \right]$$

and from the well-known trigonometric relationship:

$$\sin A - \sin B = 2 \sin \frac{1}{2}\,(A - B) \times \cos \frac{1}{2}\,(A + B)$$

it is found that:

$$\underline{F\,(t) = 2\,\Delta f \frac{\sin\,[\pi\,\Delta f\,(t - t_L)]}{\pi\,\Delta f\,t - t_L)} \times \cos\,(2\,\pi\,f_o\,t)}$$

which is the expression given in the text.

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Brief Communications

The intention of this section in the B & K Technical Reviews is to cover more practical aspects of the use of Brüel & Kjær instruments. It is meant to be an "open forum" for communication between the readers of the Review and our development and application laboratories. We therefore invite you to contribute to this communication whenever you have solved a measurement problem that you think may be of general interest to users of B & K equipment. The only restriction to contributions is that they should be as short as possible and preferably no longer than 3 typewritten pages (A 4).

Important Changes to the Telephone Transmission Measuring System

by

Ronald Walford

In 1964 the first model of the Electroacoustic Transmission Measuring System was released from the factory. It was given the type number 3350. Now, six years later, large numbers of 3350s are being used in nearly 40 countries.

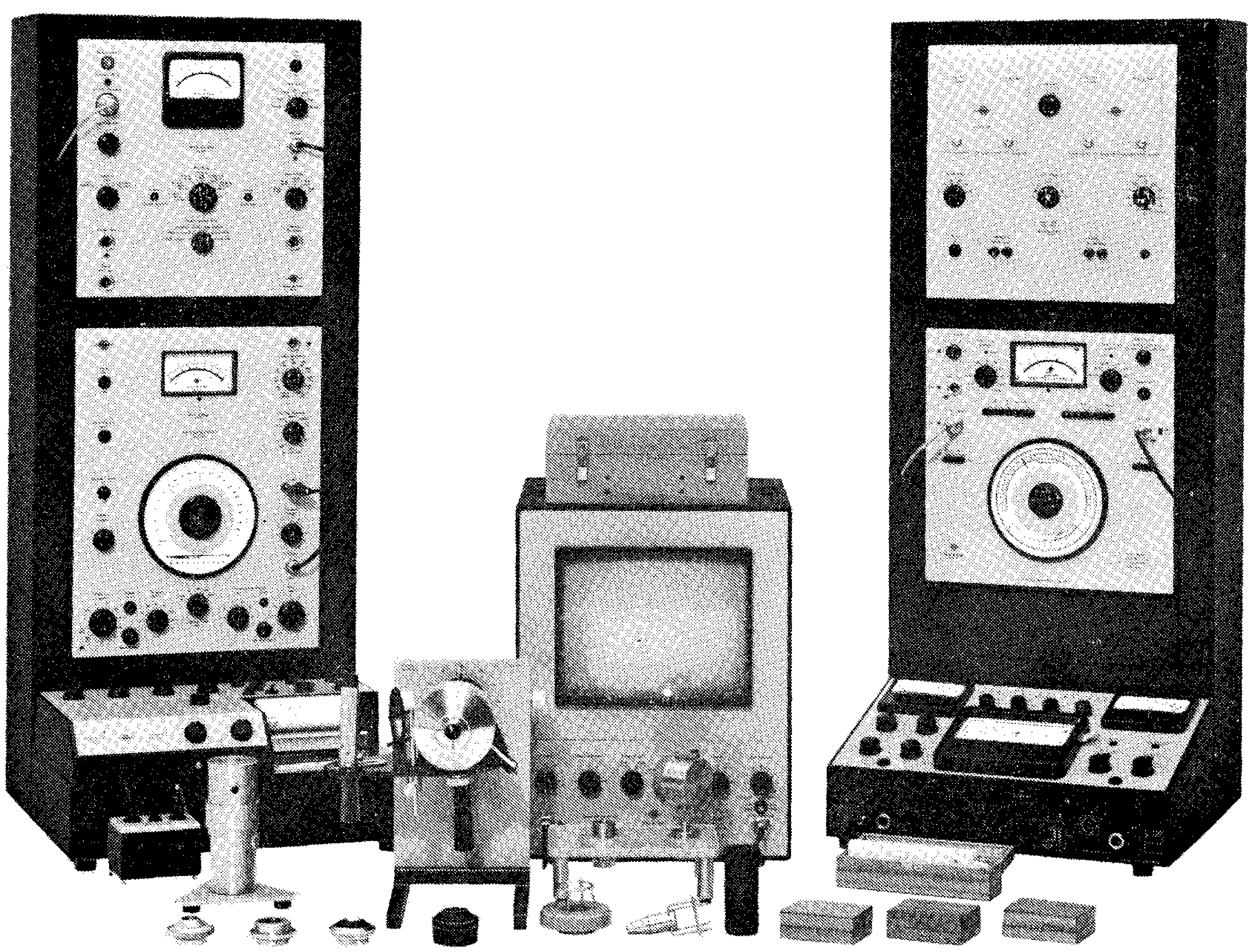


Fig. 1. The new Electroacoustic Telephone Transmission Measuring System.

During the six years of its existence there has been a marked change in the attitude of telephone engineers towards acoustic measurements. There has been a quite noticeable sharpening of interest in the use of acoustics for designing and testing telephone systems, coupled with a demand for increased accuracy of measurement. Perhaps it is too soon to say that all telephone engineers have learnt to appreciate the value of acoustic measurements, but it is certain that many of them are now performing regular acoustic tests with an accuracy and repeatability that were almost unattainable some years ago. Coupled with this, encouraging progress is being made in the clarification of test standards so that manufacturers and administrations all over the world can begin to exchange measurements with some hope that they may have a definite practical use.

We would like to think that the 3350 has played some part in bringing about this encouraging trend. We have been encouraged in this belief by the comments of the many users of the 3350. In particular we have noticed that more and more groups are beginning to build complete test standards around the 3350 with a consequently increased need for an extremely high level of performance from the equipment.

We have decided to meet this need by introducing some important changes in the design of the 3350. A new model has been built, designated the 3352, incorporating many detailed improvements and additions to the basic 3350 system.

There is no change in the basic working principle, and the same general combination of instruments is used. But the general specification has been considerably tightened, the calibration method has been improved and simplified, the stability of the whole equipment is now extremely high, standard tests have been made easier to perform, and many minor improvements have been made to the details of operation.

Additionally, a completely new range of circuits has been added to meet the expected requirements of American usage. The calibration of the new Artificial Voice conforms to the recommendations of IEEE 269 and the new Test Head performs the conditioning motion suggested by this new standard. A new integrating law for the meter uses an exponent of 0.45 and a new calibrating technique for the whole equipment brings it into line with current American thought on the measurement of loudness.

The full circuits for measurement of OREM.A and OREM.B (i.e. the two European standards) have been completely re-designed in solid-state form. A new calibrating technique considerably narrows the range of error. New accessories continue the process of reducing error margins and simplifying standard tests.

In particular, a completely new Test Head has been produced, fitted with a new Artificial Voice and a range of four Artificial Ear couplers. The combination allows telephone handsets to be gripped precisely and firmly in the REF or AEN modal positions for measurements of OREM.A, B, C and US. For

OREM.C the IEC audiometric ear has been added to the equipment.
The new Voice incorporates a built-in preamplifier for the 1/4-inch compressing microphone plus a greatly-improved mechanical construction which allows the Voice to be positively located on the Head without the use of a template.

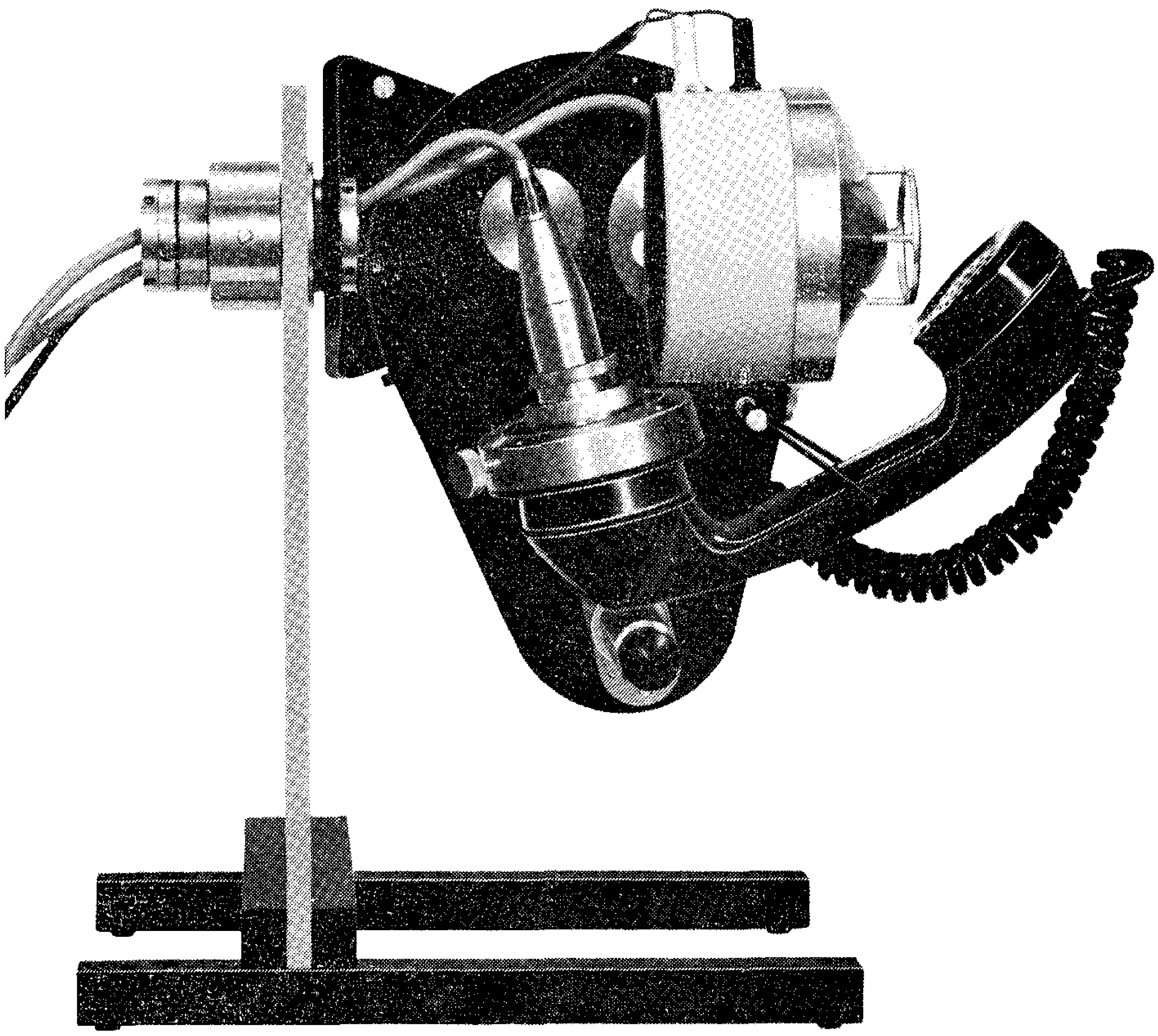


Fig. 2. The Test Head, set to the American test position. The axis of rotation coincides with the principal axis of the Voice, and the Voice lipring is set to the AEN modal position.

There are many other detailed changes to the main units and their accessories: balanced input/output circuits for the main meter; overload indicator; simplified back-panel controls; solid-state meter circuits with extremely accurate integrating laws; a new fast-rise slow-fall meter damping; a solid-state oscillator with two loudness sweep ranges; the new 2113 solid-state Spectrometer; and the new solid-state 2305 Recorder. Full details can be found in the relevant Product Data sheets.
The new 3352 will be available for delivery late in 1970.

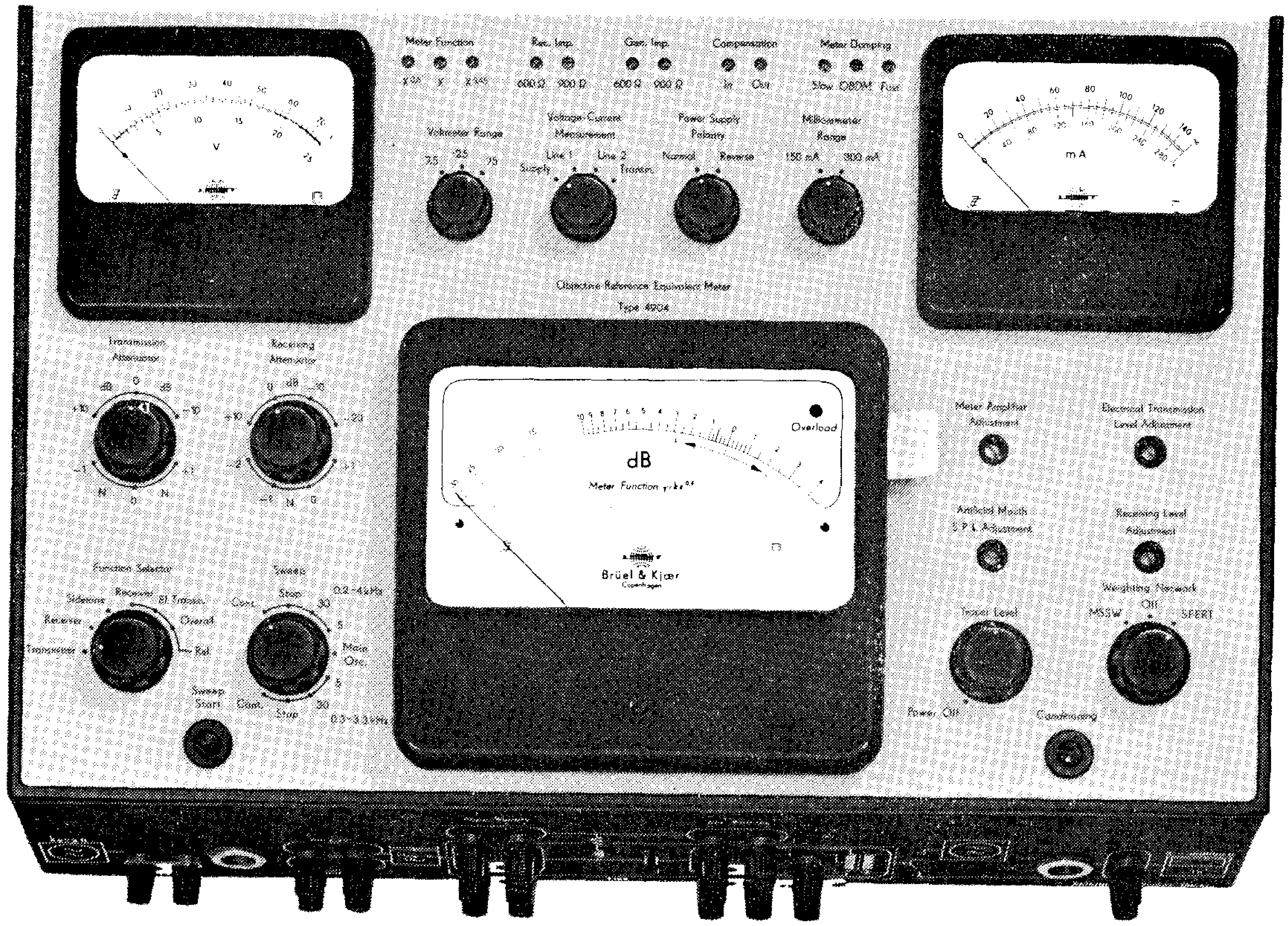


Fig. 3. The top panel of the 4904 Objective Reference Equivalent Meter. The top row of indicator lamps refers to switches on the back panel.

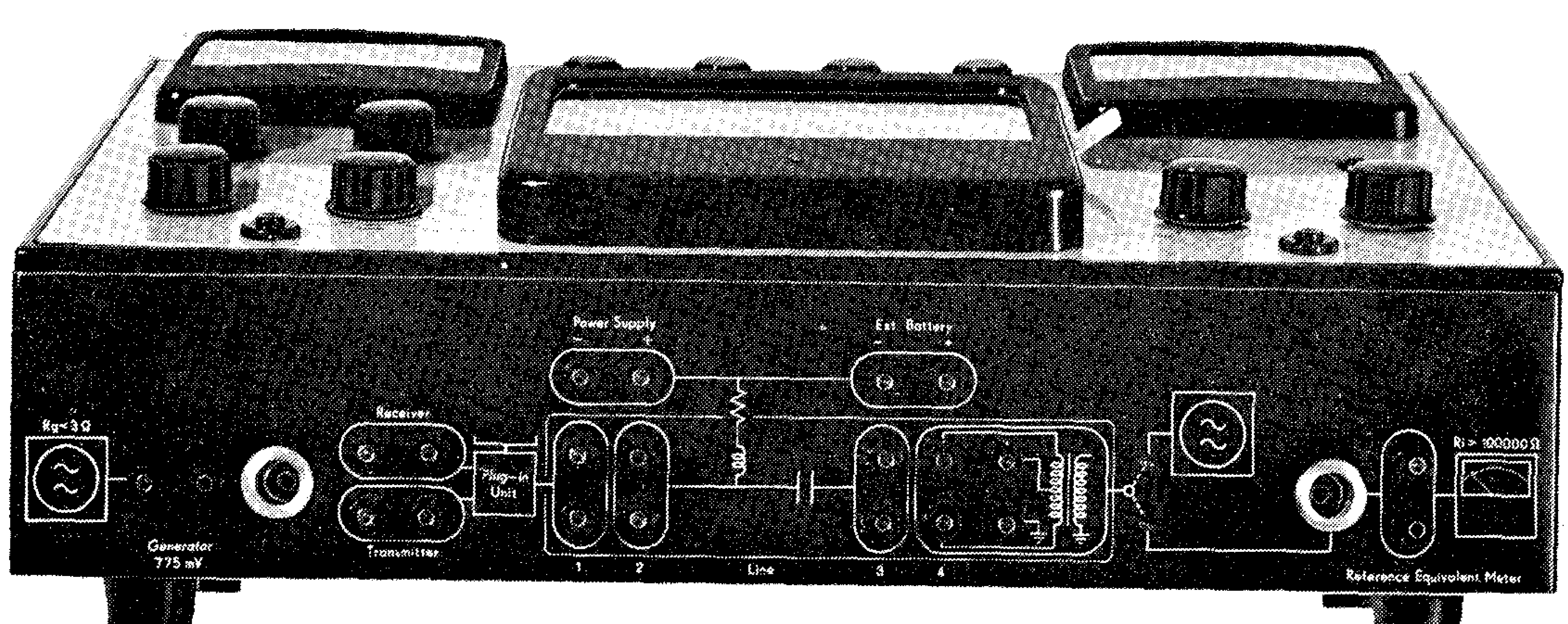


Fig. 4. The front panel of the Meter. The mimic diagram shows the balanced input/output arrangements.

News from the Factory

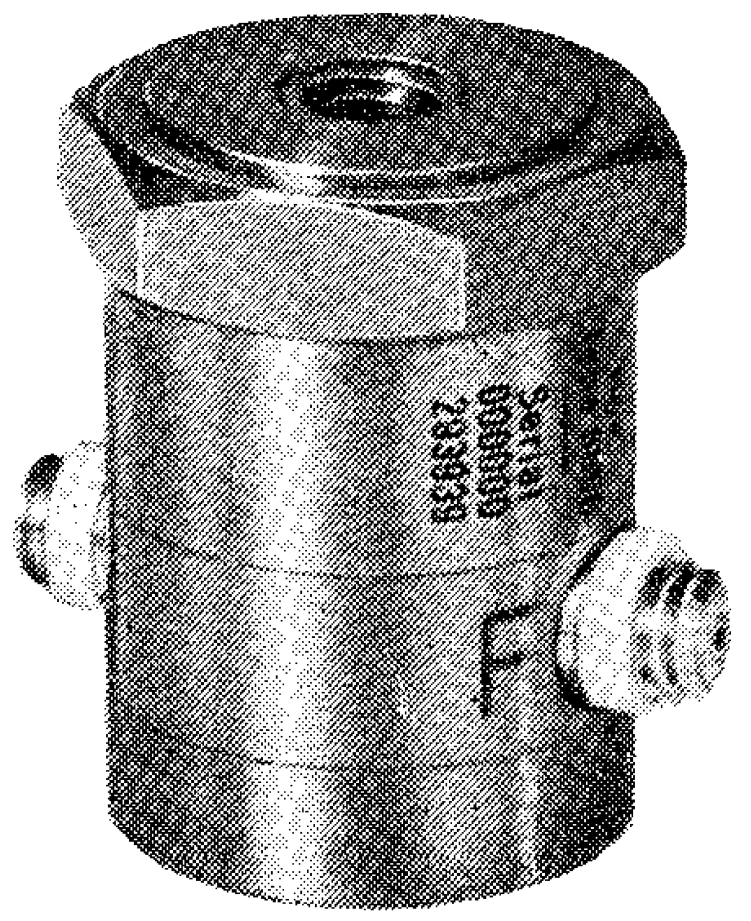
Impedance Heads Type 8000 and Type 8001

The two Impedance Heads are intended for mechanical impedance measurements in conjunction with the Mini Shaker Type 4810. As mechanical impedance is defined as force divided by velocity, the Impedance Heads, each consists of a force transducer and an accelerometer in a common Titanium housing. The driving platform of the Impedance Heads, however, is made of Beryium in order to achieve minimum mass below the force gauge – it is only 1.1 gram and 1 gram for the Type 8000 and Type 8001 respectively. Both types have a useful frequency range from almost DC to 10 kHz, their low frequency performance only limited by the preamplifier used. Until 10 kHz the phase change between the force gauge and the accelerometer base is less than 1°.

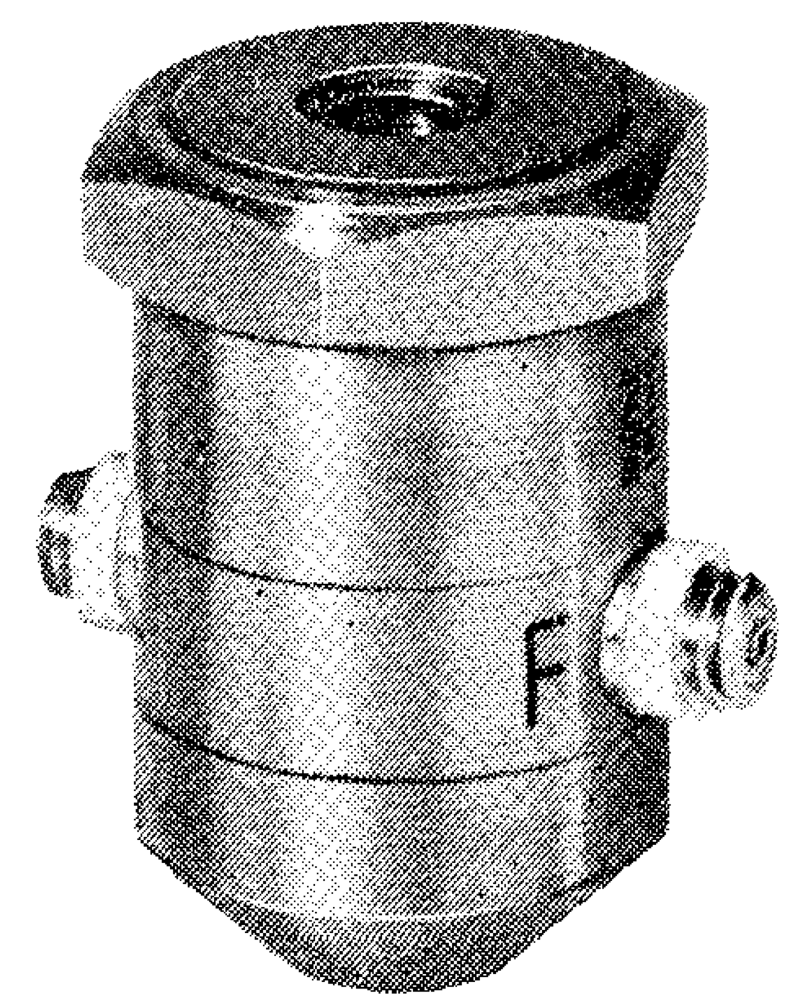
Type 8000 is intended for the calibration of the Artificial Mastoid Type 4930. It permits the measurement of mechanical impedance of other soft materials. In the medical field this facilitates mechanical impedance measurements of the human body.

Type 8001 is a general purpose device and it is very well suited for measurements on structures.

Both devices have a force sensitivity of approx. 280 mV/N and an acceleration sensitivity of approx. 25 mV/g.



Type 8000



Type 8001

Correction to the Article:
"Measurement of the Complex Modulus of Elasticity of Fibres and Folios"
B & K Techn. Rev. No. 2-1970

The deviation $d_F - d_D$ which is shown in Fig. 6 is actually caused by the mathematical approximation of equation VI on p. 10.

When exact calculations are made the deviation, $d_F - d_D$, is negligible even for large damping ratios. The curve (Fig. 6) is therefore superfluous and should be ignored.

Similarly, the η^2-axis in Fig. 5 refers to the force-excited system *only.*

B. Stisen.

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(Continued on cover page 2)

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